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PROBLEM FOR A TWO-DIMENSIONAL WAVE EQUATION IN A RECTANGLE

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Annotation: This paper investigates the solution of the problem for two-dimensional wave equation in a rectangle.
Key words: wave equation, rectangular membrane, boundary conditions, initial conditions, Fourier method, interval, boundary value problem, particular solution.

It is known that the Fourier method, based on the separation of variables, is one of the main methods of the theory of partial differential equations. When applying this method to solving a particular problem for partial differential equations with two independent variables. The desired solution to the problem $U(x,t)$ under study is sought in the form of two functions, i.e. in the form $U(x,t)=X(x)T(t)$. Substituting this kind functions $U(x,t)$ into the equation and the conditions of the problem, and then, having performed some transformations, the given problem is replaced by two problems regarding the functions $X(x)$ and $T(t)$. From this, it follows that the Fourier method is applicable only to those problems that allow separation of variables, i.e. to problems that split into two problems with respect to functions of one variable.
Let us consider small oscillations of a homogeneous rectangular membrane with sides p and q fixed along the contour
This problem is reduced to solving the wave equation

$$\frac{\partial^2 u}{\partial t^2} = a^2 \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \qquad (1)$$

under boundary conditions:

$$\begin{aligned} u\big|_{x=0} &= 0, \qquad u\big|_{x=p} = 0, \\ u\big|_{y=0} &= 0, \qquad u\big|_{y=q} = 0 \end{aligned} \qquad (2)$$

and initial conditions:

$$u\big|_{t=0} = f(x,y), \ \frac{\partial u}{\partial t}\bigg|_{t=0} = F(x,y). \qquad (3)$$

We will look for particular solutions of equation (1) in the form

$$u(x,y,t) = T(t)\upsilon(x,y), \qquad (4)$$

satisfying the boundary conditions (2).

Substituting (4) into equation (1), we obtain

$$\frac{T''(t)}{a^2 T(t)} = \frac{\upsilon_{xx} + \upsilon_{yy}}{\upsilon}.$$

It is obvious that this equality can only take place if both of its parts are equal to the same constant value. Let us denote this constant by $-\ k^2$ and, taking into account the boundary conditions (2), we find that

$$T''(t)+(ak)^2T(t)=0 \tag{5}$$

$$\upsilon_{xx}+\upsilon_{yy}+k^2\upsilon=0, \tag{6}$$

$$\begin{aligned} u\big|_{x=0}=0,\quad u\big|_{x=p}=0,\\ u\big|_{y=0}=0,\quad u\big|_{y=q}=0 \end{aligned} \tag{7}$$

We will solve the boundary value problem (6), (7) using the Fourier method,

$$\upsilon(x,y)=X(x)Y(y). \tag{8}$$

Substituting (8) into equation (6), we obtain

$$\frac{Y''(y)}{Y(y)}+k^2=-\frac{X''(x)}{X(x)},$$

from which we obtain two equations:

$$X''(x)+k_1^2X(x)=0, \qquad Y''(y)+k_2^2Y(y)=0, \tag{9}$$

Where

$$k_2^2=k^2-k_1^2 \qquad \text{or} \qquad k^2=k_1^2+k_2^2. \tag{10}$$

General solutions of equations (9), as is known, have the following form:

$$\begin{aligned} X(x)=C_1\cos k_1x+C_2\sin k_1x,\\ Y(y)=C_3\cos k_2y+C_4\sin k_2y. \end{aligned} \tag{11}$$

From the boundary conditions (7) we obtain

$$\begin{aligned} X(0)=0,\quad X(p)=0,\\ Y(0)=0,\quad Y(q)=0, \end{aligned} \tag{12}$$

from which it is clear that $C_1=C_3=0$, if we put it $C_2=C_4=1$, then it turns out:

$$X(x)=\sin k_1x, \qquad Y(y)=\cos k_2y, \tag{13}$$

and it should be

$$\sin k_1p=0, \qquad \sin k_2q=0. \tag{14}$$

From equations (14) it follows that k_1 and k_2 have countless meanings:

$$k_{1m}=\frac{m\pi}{p}, \qquad k_{2m}=\frac{n\pi}{q} \qquad (m,n=1,2,3,...).$$

Then from equality (2.1.10) we obtain the corresponding values of the constant k^2:

$$k_{mn}^2=k_{1m}^2+k_{2n}^2=\pi^2\ \frac{m^2}{p^2}+\frac{n^2}{q^2}\ . \tag{15}$$

Thus, the eigenvalues (2.1.15) correspond to the eigenfunctions

$$\upsilon_{mn}(x,y)=\sin\frac{mnx}{p}\sin\frac{n\pi y}{q} \tag{16}$$

boundary value problem (6), (7).

Turning now to equation (5), we see that for each eigenvalue its $k^2=k_{mn}^2$ general solution has the form:

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$$T_{mn}(t) = A_{mn}\cos ak_{mn}t + B_{mn}\sin ak_{mn}t. \qquad (17)$$

Thus, by virtue of (4), (16) and (17), the particular solutions of equation (1) satisfying the boundary conditions (2) have the form:

$$u_{mn}(x,y) = \left(A_{mn}\cos ak_{mn}t + B_{mn}\sin ak_{mn}t\right)\sin\frac{m\pi x}{p}\sin\frac{n\pi y}{q}. \qquad (18)$$

To satisfy the initial conditions (3), we compose a series

$$u(x,y,t) = \sum_{m=1}\sum_{n=1}\left(A_{mn}\cos ak_{mn}t + B_{mn}\sin ak_{mn}t\right)\sin\frac{mnx}{p}\sin\frac{n\pi y}{q}. \qquad (19)$$

If this series converges uniformly, as do the series obtained from it by double term-by-term differentiation with respect to x, y and t then its sum will obviously satisfy equation (1) and boundary conditions (2). To satisfy the initial conditions (3), it is necessary that

$$u\Big|_{t=0} = f(x,y) = \sum_{m=1}\sum_{n=1} A_{mn}\sin\frac{mnx}{p}\sin\frac{n\pi y}{q}, \qquad (20)$$

$$\frac{\partial u}{\partial t}\bigg|_{t=0} = F(x,y) = \sum_{m=1}\sum_{n=1} ak_{mn}B_{mn}\sin\frac{mnx}{p}\sin\frac{n\pi y}{q}. \qquad (21)$$

Assuming that the series (20) and (21) converge uniformly, we can determine the coefficients A_{mn} and B_{mn}, multiplying both parts of equalities (20) and (21) by

$$\sin\frac{m_1\pi x}{p}\sin\frac{n_1\pi y}{q}$$

and integrating over x the interval from 0 to p and over y 0 to q. Then, taking into account that

$$\int_0^p\int_0^q \sin\frac{m\pi x}{p}\sin\frac{npy}{q}\sin\frac{m_1\pi x}{p}\sin\frac{n_1 py}{q}dxdy =$$

$$= \begin{cases} 0, & \text{если } m \neq m_1, \\ & \text{или } n \neq n_1, \\ \dfrac{pq}{4}, & \text{если } m_1 = m, \quad n_1 = n, \end{cases}$$

we will receive

$$A_{mn} = \frac{4}{pq}\int_0^p\int_0^q f(x,y)\sin\frac{m\pi x}{p}\sin\frac{n\pi y}{p}dxdy.$$
$$(22)$$
$$B_{mn} = \frac{4}{apqk_{mn}}\int_0^p\int_0^q F(x,y)\sin\frac{m\pi x}{p}\sin\frac{n\pi y}{q}dxdy.$$

Solution (19) can also be written in the form

$$u(x,y,t) = \sum_{m=1}\sum_{n=1} M_{mn}\sin\frac{m\pi x}{p}\sin\frac{n\pi y}{q}\sin\left(ak_{mn}t + \varphi_{mn}\right), \qquad (23)$$

Where

$$M_{mn} = \sqrt{A_{mn}^2 + B_{mn}^2}, \qquad \varphi_{mn} = arctg\left(A_{mn} / B_{mn}\right).$$

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