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THE PROBLEM OF DETERMINING FUNCTIONS BY SPHERICAL AVERAGE VALUE

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Abstract: This article considers the general concept of weakly incorrectness. Weakly incorrectness refers to problems for which it is possible to obtain conditional stability estimates of the level type. As a result, we consider many important moments in checking the integral geometry problem. This is, firstly, the question of the uniqueness of the solution of the equation in some class of functions. Secondly, obtaining a stability estimate in solving the problem. We know that a wider class of integral geometry problems is considered to be strongly incorrect and has a conditional stability estimate with a larger value.

Keywords: Fourier transform, uniqueness theorem, strongly ill-posed problems, inversion theorem .

As a problem of integral geometry $u(x)$ function with respect to a definite integral, i.e. $f(y)$ It is said to be a matter of restoration by function.

The definition of M.M.Lavrentev , an integral geometry problem is called a Voltaire-type integral geometry problem if the problem can be reduced to the study of Voltaire-type operator equations. $U(x)-R^n$ is a fairly smooth function and $\{S(y)\}-y=(y_1,y_2,...,y_k)$ Let there be a family of smooth polynomials in this space depending on the parameter . Suppose

$$\int_{S(y)} g(x,y)u(x)ds = f(y) \quad (1)$$

Let the integral be known, here $g(x,y)$ The given weight functions $ds-S(y)$ determine the element in the scale.

We also give a definition of strong and weakly irregular integral geometry problems.

If

$$\int_{S(y)} g(x,y)u(x)ds = f(y) \quad (1)$$

If a pair of functional spaces is found for the givens and solutions of the problem, and the inversion operator for this pair of spaces is continuous and a finite number of derivatives participate in the norm determination, then the problem of solving equation (1) is called **a weakly ill-posed problem** , and if no such pair of spaces exists, it is called **a strongly ill-posed problem** .

Such classes of M.M.Lavrentev are relevant not only for the problem of integral geometry, but also for the general theory of ill-posed problems.

However, this is only possible in special cases. This rule is only valid for those spaces where these manifolds and weight functions are sufficiently rich in automorphism groups and are invariant to themselves. As a result, the feasibility problem, represented in the form (1), is $f(y)$ It consists of finding some necessary and sufficient conditions under which the function can be satisfied.

Many constructions of the theory of ill-posed problems and operator equations and their numerous applications are described in the monographs and articles of A.N.Tikhonov, M.M.Lavrentev, and V.K.Ivanov.

$n-$measurable $(x_1,...,x_n)$ Spheres of arbitrary radius ( ) $n=2$ passing $0<r<\infty$ through a set of plane points with a center assigned to them r in space ($n \geq 2$ (in circles) $u(x_1,...,x_n)$ Let us

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consider the problem of finding functions by their mean values [1]. For convenience, we will use this plane $x_n = 0$ We superimpose it on the coordinate plane and the space is arbitrary $(x_1,...,x_n)$ point (x,y) We define by , where $x=(x_1,...,x_{n-1})$. Thus $u=u(x,y)$. The problem stated in these definitions is $u(x,y)$ function of its

$$\frac{1}{\omega_n}\int_{S(x,r)} u(\xi,\eta)d\omega = \upsilon(x,r) \qquad (2)$$

by integrals .
Here $(\xi,\eta)\neg S(x,r)$ a variable point on a sphere, $\xi=(\xi_1,...\xi_{n-1})$, ω_n is $n-$ the surface of a unit sphere in dimensional space:

$$\omega_n = \frac{2\pi^{n/2}}{\Gamma(n/2)}$$

$d\omega$ -surface element, dS with which $dS = r^{n-2}d\omega$ the telescoping angle element is connected by the formula.

Ravshanki, y An arbitrary odd function in the variable (2) $\upsilon(x,r)=0$ is a solution of the homogeneous equation, that is, the equation. Therefore $\upsilon(x,r)$ only the problem of determining by function $u(x,y)$ the even part of the function, namely:

$$u_1(x,y) = \frac{1}{2}[u(x,y)+u(x_1-y)]$$

the problem of defining the function . In this y case, a pair $u(x,y)$ is as powerful as looking at the class of functions. Obviously, such a setting

$$u(x,y)=0\,,\, y \quad 0$$

that satisfies the condition .
The following uniqueness theorem holds.

1.Theorem. $D(x_0,r_0)=\{(x,y):|x-x_0|^2+y^2<r_0^2\}$ continuous and y An arbitrary $u(x,y)$ function on the domain $G_\varepsilon(x_0,r)=\{(x,r):|x-x_0|<\varepsilon,\ 0<r<r_0-|x-x_0|\}$, where ε is a positive number, $\varepsilon \quad r_0$ is defined as a single-valued function given by ;

The proof of the theorem is given by each assigned $S(x,r)$, $(x,r) \quad G_\varepsilon(x_0,r_0)$ in the sphere x It is based on calculating all the moments of a function corresponding to a variable $u(x,y)$. $G_\varepsilon(x_0,r_0)$ The sphere resembles a very sharp pencil (Figure 1). $\varepsilon = r_0$ This " pen" rotates until it becomes a cone (Figure 2).

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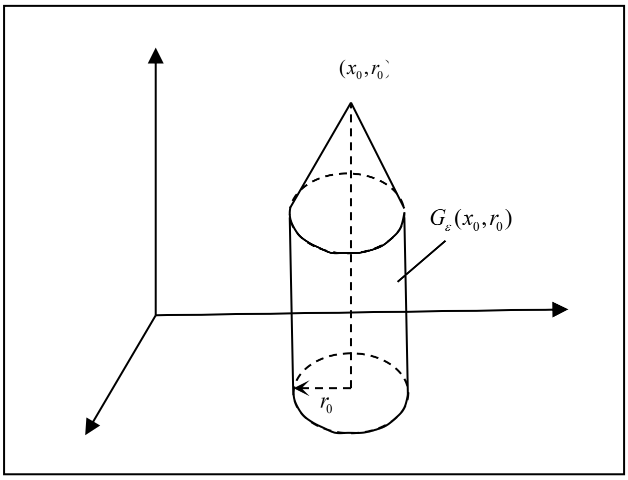

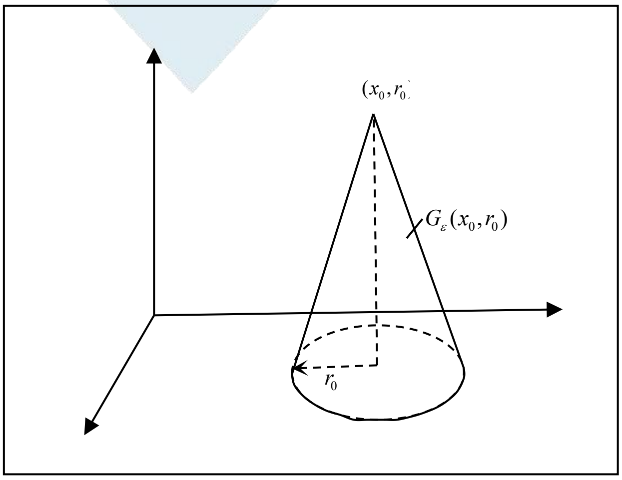

Picture 1 Picture 2

Optional (x,r) $G_\varepsilon(x_0,r_0)$, $0<\varepsilon$ r_0 for the point $S(x,r)$ the sphere lies entirely within the sphere.

$$A_i\upsilon=\frac{\partial}{\partial x_i}{}_0^r\,\tau^{n-1}\upsilon(x,\tau)d\tau,\qquad i=1,\dots,n-1$$

the operator to equation (2). By performing a series of substitutions,

$$A_i\upsilon=\frac{1}{\omega_n}\frac{\partial}{\partial x_i}{}_0^r\,\tau^{n-1}\;{}_{S(x,r)}\,u(\xi,\eta)d\omega\;d\tau=$$

$$=\frac{1}{\omega_n}\frac{\partial}{\partial x_i}\;{}_{|\xi-x|_i^2+\eta^2\;\;r^2}\;{}_{x_i-\left[r^2-\eta^2-|\xi-x|_i^2\right]^{1/2}}^{x_i+\left[r^2-\eta^2-|\xi-x|_i^2\right]^{1/2}}\,d\eta d\xi_1\dots d\xi_{i-1}\dots d\xi_{i+1}\dots d\xi_{n-1}=$$

$$=\frac{1}{\omega_n}\;{}_{S(x,r)}\,u(\xi,\eta)\cos(n,\xi_i)dS=\frac{r^{n-2}}{\omega_n}\;{}_{S(x,r)}\,u(\xi,\eta)(\xi_i-x_i)d\omega,$$

We find . Here in the intermediate calculations $|\xi-x|_i$ through $(\xi,0)$, the expression $(x,0)$ that is numerically equivalent to the projection onto the plane $\left[|\xi-x|^2-|\xi_i-x_i|^2\right]^{1/2}$ of $y=0$ the straight line segment connecting the points $x_i=0$, n through $S(x,r)$ The unit vector of the external normal to is defined.

Now L_i through

$$L_i\upsilon=\frac{1}{r^{n-2}}A_i\upsilon+x_i\upsilon(x,r),\quad i=1,\dots,n-1,$$

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From the formula defining the operator defined by equality, $u_i(x,y) = x_i u(x,y)$ we find that its application to equation (2) is as powerful as calculating the spherical mean of the function.

Indeed

$$L_i\upsilon = \frac{1}{\omega_n} \int_{S(x,r)} u(\xi,\eta)\xi_i d\omega$$

understandably, L_j L_i superposition of operators $\upsilon(x,y)$ result of applying to a function $u(x,y)x_i x_j$ is as powerful as calculating the spherical mean of the function. In general, if is a polynomial of degree -th with $P_m(x)$ constant coefficients m, then $P_m(L) L = (L_1,...,L_m)$ Application of operator (2) to equation

$$P_m(L)\upsilon = \frac{1}{\omega_n} \int_{S(x,r)} u(\xi,\eta) P_m(\xi) d\omega$$

to equality .

Now (x,r) Let $S(x,r)$ the equality obtained on the sphere be $D_0(x,r) = \{\xi : |\xi - x| \quad r\}$

$$P_m(L)\upsilon = \int_{D_0(x,r)} \varphi(\xi,x,r) P_m(\xi) d\xi \qquad (3)$$

can be written as, where $\varphi(\xi,x,r)$ the function (x,r is assigned) $u(x,y)$ with the function as follows

$$\varphi(\xi,x,r) = \frac{2}{\omega_n r^{n-2}} \frac{u\left(\xi,\left[r^2 - |x-\xi|^2\right]^{1/2}\right)}{\left[r^2 - |x-\xi|^2\right]^{1/2}}$$

by formula .

In this $u(x,y)$ The function's y duality with respect to the variable is taken into account.

Various $P_m(\xi)$ with a system of equations (3) corresponding to polynomials $\varphi(\xi,x,r)$ function and $u(x,y)$ The function is also defined as single-valued.

The theorem has been proven . $u(x,y)$ to construct $P_m(\xi)$ a function by function constructively $\upsilon(x,r)$ as $D_0(x,r)$ In this case, we can obtain a system of orthogonal polynomials. $\varphi(\xi,x,r)$ The function can be written in explicit form using a Fourier series. This $u(x,y)$ requires placing more conditions on the function to ensure that the Fourier series converges.

1. Result. If $u(x,y)$ function $D(x_0,r_0)$ If the domain $u_y(x,y)$ is continuous together with the particular derivative, then it is also continuous between $D(x_0,r_0)$ the $u(x,y)$ function itself and its $u_y(x,y)$ all possible from the private property $S(x,r)$, (x,r) $G_\varepsilon(x_0,r_0)$, $0 < \varepsilon \quad r_0$ is determined by giving the spherical mean values over the spheres.

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In fact, in this case, the even part $u(x,y)$ of the function y with respect to the variable and $u_y(x,y)$ The even part of the particular derivative is defined as a single value. However, an arbitrary $u(x,y)$ function from the class under consideration

$$u(x,y)=\frac{1}{2}\left[u(x,y)+u(x,-y)\right]+\frac{1}{2}\int_0^y\left[u_y(x,y)+u_y(x,-y)\right]dy$$

can be expressed in terms of and therefore can be found to have a single value.
It follows from the proved theorem that, $\upsilon(x,r)$ function with a sufficiently small positive to be given in the $u(x,y)$ field $G_\varepsilon(x_0,r_0)$ defines the function $D(x_0,r_0)$ in the domain. However, $u(x,y)$ function $D(x_0,r_0)$ knowing in the field, $D(x_0,r_0)$ Integrals can be calculated over all possible spheres that lie entirely within the sphere, $\varepsilon=r_0$ so $\upsilon(x,r)$ for (x,r) $G_\varepsilon(x_0,r_0)$ function can be found. Thus $\upsilon(x,r)$ Given $G_{r_0}(x_0,r_0)$ $G_\varepsilon(x_0,r_0)$ a function in a domain, it $G_\varepsilon(x_0,r_0)$ is defined as a single value in a larger domain.
This means that, $\upsilon(x,r)$ The function may not be given arbitrarily. In addition, the arbitrary function given in the form (2) $\upsilon(x,r)$ has a property similar to that of analytic functions: it can be defined $G_{r_0}(x_0,r_0)$ in a single-valued manner in an arbitrarily narrow $G_\varepsilon(x_0,r_0)$ domain with values of . At the same time, it is clearly not analytic $u(x,y)$ functions correspond to $\upsilon(x,r)$ non-analytic functions. According to the above $\upsilon(x,r)$, the constructive description of the class of functions expressed in the form (2) is a rather difficult problem. Now we show that the problem of solving equation (2) is classically ill-posed, that is, it $\upsilon(x,r)$ is strongly unstable with respect to small changes in the function. This is easy to do for three-dimensional space, so in the following we $n=3$ will assume that . Moreover, here we assume that in the entire (x,y) space $u(x,y)$ function twice times continuously differentiable and y We assume that the variable is even.
We introduce the following function

$$w(x,y,r)=\frac{r}{\omega_3}\int_{(\xi-x)^2+(\eta-y)^2=r^2}u(\xi,\eta)d\omega \qquad (4)$$

she is r center of gravity (x,y) on point r is the spherical mean value over a sphere of radius . It is known from the mathematical physics course that, $w(x,y,r)$ function in half space

$$\partial^2 w/\partial r^2=\Delta_{xy}w \qquad (5)$$

the wave equation . Here is $\Delta_{xy}w$ the (x,y) Laplace operator in the variables. It is immediately clear from formula (4) that

$$w(x,y,0)=0 \qquad (6)$$

(2) from

$$w(x,y,r)\big|_{y=0}=r\upsilon(x,r) \qquad (7)$$

The equality follows . Finally, $u(x,y)$ pair by function y

$$w_y(x,y,r)|_{y=0} = 0 \qquad (8)$$

to equality .
satisfying equation (5) and conditions (6)-(8) $w(x,y,r)$ is a boundary value problem, which is clear if $r<0$ in the domain $\upsilon(x,r)$ If we continue the function in pairs, (x,y,r) (5), (7), (8) are equally strong as the Cauchy problem in the entire space.

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