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PEDAGOGICAL AND PSYCHOLOGICAL FOUNDATIONS OF INTEGRATING MATHEMATICS EDUCATION

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Annotation: This article discusses the psychological and pedagogical foundations of integration in mathematics education, focusing on ideas about ensuring the coherence of scientific knowledge, as well as the thoughts and views of past thinkers. It explores the efforts in the field of scientific knowledge integration and the ideas of pedagogical scholars in our republic regarding the organization of teaching based on integrative knowledge in the natural sciences. The article also addresses the lack of mutual integration between pedagogical and methodological scholars in explaining the essence of educational integration.

Keywords: integration, perpendicular, coherence, mathematics, plane, logic.

Аннотация: В статье рассматриваются психологические и педагогические основы интеграции в математическом образовании, уделяя внимание идеям обеспечения целостности научных знаний, а также мыслям и взглядам мыслителей прошлого. Исследуются усилия в области интеграции научных знаний и идеи педагогических ученых нашей республики относительно организации обучения на основе интегративных знаний в области естественных наук. В статье также рассматривается отсутствие взаимной интеграции между педагогическими и методическими учеными в объяснении сути образовательной интеграции.

Ключевые слова: интеграция, перпендикуляр, целостность, математика, плоскость, логика.

In Uzbekistan, the education system is undergoing fundamental reforms to elevate it to a new level in line with modern times and transition to a new stage of interaction between the state and society. To achieve this, every individual living in the present era must possess sufficient knowledge for the future of the country, ensuring its comprehensive development and progress. The preparation of independent-thinking, creative specialists who can consciously apply their knowledge in life begins with school education.

According to academician N.N. Semenov, mandatory obedience in the teacher-student relationship should be replaced by free and conscious discipline. Furthermore, the main task of a teacher should be to develop students' ability to think independently and freely and to systematically enhance this skill. It is well known that in any legal and democratic state, students—and, more broadly, every member of society—should be educated to think freely, and the education system should be structured accordingly.

In particular, one of the most essential conditions for mastering mathematics is developing students' thinking abilities, as full comprehension of the subject is only possible through cognitive growth. Global market integration and the rapid pace of scientific and technological progress require strengthening interconnections between all levels of education. This, in turn, necessitates an integrated approach to teaching and learning.

In today's socio-economic conditions, the education system must align with democratic and market economy changes. It is crucial to ensure that the learning process is sufficiently supported by material, technical, and informational resources, provided with highly qualified scientific and pedagogical staff, and supplemented with high-quality textbooks, teaching methodologies, and literature. Additionally, establishing strong integration between education, science, and industry is essential.

This, of course, requires giving great attention to mathematics and its development. The laws, theoretical concepts, and practical applications of mathematics continue to evolve in response to real-world demands. The advancement of mathematics requires a comprehensive, integrative understanding of its essence, methods of cognition, and tools.

The concept of ensuring the integrative (holistic) nature of scientific mathematical knowledge is reflected in the works of past thinkers. More than 300 years ago, Euclid's famous work Elements proved theorems using geometric methods, and even arithmetic problems were explained in the language of geometry. In those times, many algebraic identities were also demonstrated through geometric constructions. This suggests that arithmetic and algebra were integrated with geometry.

Moreover, the works of our ancestors, such as Al-Khwarizmi, Al-Farabi, Ibn Sina, A.R. Beruni, and M. Ulugh Beg, in fields like mathematics, astronomy, geography, medicine, and other sciences, discuss the relationship between quantity and quality, as well as the unity of natural, social, and applied sciences.

Al-Khwarizmi's famous work, Al-Kitab al-Mukhtasar fi Hisab al-Jabr wa al-Muqabala, which holds an important place in the history of mathematics, consists of three sections:

1. The first section presents an algebraic method for solving first- and second-degree equations with a single unknown, ensuring that only non-negative roots are considered. He also provides a geometric analysis of these solutions.
2. The second section is dedicated to geometry, where algebraic applications to measurement problems and calculations related to quantities are discussed.
3. The third section explores the practical applications of algebra, particularly problems related to inheritance distribution.

In the introduction of his work, Al-Khwarizmi also provides information about the contributions of past scholars to the development of science. His book covers both simple and complex arithmetic problems and emphasizes their necessity in legal and commercial matters such as inheritance distribution, drafting wills, property division, land measurement, canal construction, and area calculations.

Abu Nasr al-Farabi, in his work The Origins of Sciences and Their Classification, describes more than 30 sciences known during the medieval period and highlights their significance. He categorizes all sciences into five groups and includes the following subjects under the mathematics category: arithmetic, geometry, optics, astronomy, music, the science of weights, and mechanics. This suggests that even in his time, Farabi pursued an integrated approach to knowledge in his writings.

Abu Ali Ibn Sina, in his Book of Knowledge, presents the foundations of planimetry and stereometry as a unified subject. For example, in the section titled On the Basics of Intersecting Lines in Stereometry, he discusses perpendicularity to a straight line and simultaneously addresses perpendicularity to a plane. Additionally, Ibn Sina describes number notation and the comparison of numerical equality and inequality. His works, such as The Canon of Medicine, Kitab al-Najat, and Kitab al-Insof, integrate knowledge from fields such as geometry, astronomy, botany, zoology, and logic.

In the field of scientific knowledge integration, scholars such as B.M. Kedrov, P.N. Fedoseev, and N.P. Devenchuk have conducted research. However, they have not provided a clear answer to what constitutes the objective basis of knowledge integration.

From a scientific perspective, the objective basis of knowledge integration lies in the unity of the universe and the interrelationships of its constituent elements. E.O. Turdiqulov defines scientific knowledge integration as follows:

“Integration is the process of uniting scattered, fragmented elements into a coherent, systematic whole. It is the synthesis of knowledge about nature into a unified body, directing knowledge from various disciplines toward a common goal. It expresses the unity of the universe” [2.198].

M.M. Mamadazimov, in his scientific article, discusses inter- and intra-disciplinary integration, stating:

“Integration in education involves moving beyond a simple connection between subjects to an advanced stage of mutual influence between different sciences, ensuring an effective interdisciplinary approach” [3.30-b].

The term integration comes from the English word integration, meaning restoration, unification, or the merging of separate elements into a whole (Encyclopedic Dictionary).

Regarding the integration of mathematical sciences in schools, research has mainly focused on:

- Establishing internal and interdisciplinary connections,
- Developing integrated course materials,
- Strengthening the practical application of knowledge,
- Achieving didactic coherence,
- Ensuring consistency in mathematical education.

For example, the connection between algebra and geometry has been established primarily by using algebraic (analytical) methods to solve geometric problems.

On the topic of interdisciplinary connections, the great didactician J.A. Comenius emphasized:

“All interconnected things must be studied in relation to one another” [4.242-b].

Later, several educators developed and expanded upon this idea. For example, I.G. Pestalozzi drew attention to the diverse interconnections among scientific disciplines, stating:

"In one's mind, all sciences that are inherently interconnected should be structured according to their natural relationships, just as they are connected in reality" [5.202-b].

The Role of Integration in the Development of Sciences

The renowned mathematician Al-Khwarizmi made significant contributions to the history of mathematics with his famous work "Al-Kitab al-Mukhtasar fi Hisab al-Jabr wa al-Muqabala". This book consists of three sections:

1. The first section presents algebraic methods for solving first- and second-degree equations with non-negative roots and includes their geometric analysis.
2. The second section is dedicated to geometry, where measurement of quantities and the application of algebra to measurement-related problems are discussed.
3. The third section explores the practical applications of algebra, particularly problems related to inheritance distribution.

Additionally, in the introduction of his book, Al-Khwarizmi provides information about the contributions of past scholars to the development of science. His work encompasses both simple and complex arithmetic problems, emphasizing their importance in inheritance division, drafting wills, property distribution in legal and commercial affairs, land measurement, canal construction, and surface measurement.

Abu Nasr al-Farabi, in his work "The Origins and Classification of Sciences", describes and highlights the significance of more than 30 sciences known during the Middle Ages. He categorizes all sciences into five groups and includes arithmetic, geometry, optics, astronomy, music, the science of weights, and mechanics within the mathematics group. This indicates that Farabi integrated knowledge in his works even in that era.

Abu Ali Ibn Sina (Avicenna), in his "Book of Knowledge", unifies the fundamentals of planimetry and stereometry. For instance, in the section "On the Principles Related to Intersecting Lines in Stereometry", he discusses perpendiculars to a straight line and also those to a plane. Moreover, Ibn Sina explores numerical notation, comparisons of equality and inequality, and logical operations on numbers. His famous works, such as "The Canon of Medicine" (Kitab al-Qanun fi al-Tibb), "The Book of Salvation" (Kitab al-Najat), and "The Book of Fairness" (Kitab al-Insaf), integrate knowledge of geometry, astronomy, botany, zoology, and logic.

Several scholars, including B.M. Kedrov, P.N. Fedoseyev, and N.P. Devenchuk, have worked on the integration of scientific knowledge but have not found a definitive answer regarding its objective foundation. From a scientific perspective, the objective basis of knowledge integration lies in the unity of the universe and the interrelation of its elements.

E.O. Turdiqulov defines the integration of scientific knowledge as follows: "Integration is the process of bringing together scattered, fragmented, and separate elements into a whole, a systematic and unified state. It represents a comprehensive understanding of nature and directs knowledge from various fields toward a single goal. It expresses the integrity of the universe."

M.M. Mamadazimov, in his research, discusses interdisciplinary and intradisciplinary integration, stating:
"Integration means transitioning from the mere interconnection of different subjects in education to a deeper mutual influence between them."

The term "integration" originates from the English word "integration", meaning restoration of parts, unification, or fusion into a whole (according to an encyclopedic dictionary).

In school mathematics, research on subject integration mainly focuses on establishing internal and interdisciplinary connections, developing integrated courses, enhancing applied approaches, ensuring didactic consistency, and maintaining continuity in mathematical education. For example, the link between algebra and geometry is primarily established by applying algebraic (analytical) methods to solve geometric problems.

In the discussion of interdisciplinary connections, the great didactician J.A. Comenius stated: "Everything interconnected should be studied in the same interconnected manner."

Later, various pedagogues further developed and generalized this idea. For example, I.G. Pestalozzi emphasized the diversity of connections between educational subjects: "In one's mind, all sciences should be connected exactly as they are interconnected in nature."

The distinguished educator K.D. Ushinsky, in his selected works (Vol. III, M., Prosvisheniye, 1969, pp. 59-60), thoroughly substantiates the didactic importance of interdisciplinary connections from both psychological and pedagogical perspectives.

The integration of algebra and geometry in education should follow this system:

- Explaining algebraic material during the learning process
- Providing an analytical proof of geometric material
- Ensuring a systematic and sequential connection between algebra and geometry

Various psychologists have also studied interdisciplinary connections. For example, I.P. Pavlov emphasized that the foundation of scientific cognition is research and observation. He argued that human brain activity relies on systematically integrated information. Psychologist I.A. Mironenko proposed the idea of shaping a bio-social character in individuals, considering it one of the main challenges in modern school education. According to Mironenko, today's school curriculum lacks integration between worldly sciences, language, and the spiritual teachings of great individuals, which should not remain isolated.

L. S. Vygotsky highlighted the role of global sciences in shaping human psychology and the necessity of their full integration, stating:

"One of the greatest difficulties an adolescent must overcome at the end of their transitional period is transferring the meaning and significance of a developed concept to new concrete situations, which they initially think about in an abstract manner."

N.A. Menchinskaya, in her book "Psychology of Teaching Arithmetic", asserts that students' intellectual development is characterized not only by the volume of acquired knowledge but also

by their ability to apply cognitive methods and operations in problem-solving. She believes that a key indicator of intellectual growth is a person's ability to transfer their cognitive skills from one area to another.

The educational reformer and American pedagogue John Dewey introduced a new principle for structuring curricula and textbooks:

"From the child to the world and from the world to the child."

Dewey is considered one of the founders of integrated education. In his article "Integration of Subjects in the Modern School", T.G. Braje states:

"The origin of integration is a pedagogical phenomenon... The current process of integration is based on a long history of establishing interdisciplinary connections."

The author classifies the stages of educational integration into the following directions:

- The emergence of integrated sciences, reflecting the relationships between humans and nature, humans and society, history and geography, and other disciplines.
- The integration of "classical" subjects, such as history and literature, Russian language and Russian literature, and world literature with Russian literature and art. This leads to the introduction of interdisciplinary lessons covering topics relevant to multiple courses.
- Intra-disciplinary integration, where different branches within a subject are unified into a single course, restructuring existing disciplines.

Many scholars believe that integrating subjects and improving curricula remains a crucial tool for advancing educational systems.

An analysis of pedagogical and psychological literature reveals that there is no single, universally accepted definition of integrative education. However, all educators and psychologists acknowledge the effectiveness of integrative learning and its alignment with modern educational requirements. To clarify the essence of educational integration, consensus and mutual understanding among pedagogues and methodological scholars are necessary. This indicates that the issue of integration remains unresolved to some extent.

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