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SCALAR, VECTOR AND MIXED PRODUCT OF VECTORS

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Annotation: This article provides insights into the scalar product of vectors,the vector product,and the mixed product, showing the ways in which the problems involved are solved.Methods for solving several problems using properties of vectors have been recommended.

Keywords: vector, scalar product,vector product,mixed product, formula.

VEKTORLARNING SKALYAR,VEKTOR VA ARALASH KO`PAYTMASI

Annotatsiya: Ushbu maqolada vektorlarning skalyar ko`paytmasi,vektor ko`paytmasi va aralash ko`paytmasi haqida tushunchalar berilib,ularga doir masalalarning yechilish usullari ko`rsatib berilgan.Bir necha masalalarni vektorlarning xossalaridan foydalangan holda yechish metodlari tavsiya qilingan.

Kalit so'zlar:vektor,skalyar ko`paytma, vektor ko`paytma,aralash ko`paytma,formula.

KIRISH

Vektorga oid masalalar o`quvchilarning bilimlarini oshirish bilan birgalikda algebraik,geometrik masalalarni qulay usuldagi yechimlarini topishga yordam beradi.Jumladan vektorlar yordamida trigonometrik va boshqa ko`rinishdagi funksiyalarning qiymatlar to`plamini topish, geometrik jismlarning hajmlarini topish kabi masalalarni yechishda qulaylik yaratadi.Shu bilan birga o`quvchilarning mantiqiy fikrlashlarini rivojlantiradi,o`quvchilarning qiziqishlarini oshiradi.

**I.Vektorlarning skalyar ko`paytmasi.** $\vec{a}$ vektorning koordinatasi $(x_1; y_1; z_1)$ va $\vec{b}$ vektorning koordinatasi $(x_2; y_2; z_2)$ bo`lsin.U holda $\vec{a}$

va $\vec{b}$ vektorlarning skalyar ko`paytmasi deb $\vec{a} \cdot \vec{b} = x_1x_2 + y_1y_2 + z_1z_2$ songa aytiladi.

1-misol. $\vec{a}(2; -3; 9)$ va $\vec{b}(7; -8; -10)$ vektorlarning skalyar ko`paytmasini toping.

Yechish.$\vec{a} \cdot \vec{b} = x_1x_2 + y_1y_2 + z_1z_2 = 14 + 24 - 90 = -52.$

Demak, $\vec{a}(2; -3; 9)$ va $\vec{b}(7; -8; -10)$ vektorlarning skalyar ko`paytmasi -52 ga teng.

2-misol. $\vec{a}(4; -6; 9)$ va $\vec{b}(2; -2; m)$ vektorlarning skalyar ko`paytmasi 47 ga teng bo`lsa,m ni toping.

Yechish.

$$\vec{a} \cdot \vec{b} = x_1x_2 + y_1y_2 + z_1z_2 = 8 + 12 + 9m = 47.$$

Bu tenglikdan m =3 ekanligi kelib chiqadi.

Agar $\vec{a}$ va $\vec{b}$ vektorlar perpendikulyar bo`lsa,skalyar ko`paytma nolga teng bo`ladi.

3-misol. $\vec{a}(3; 6)$ va $\vec{b}(k; -3)$ vektorlar perpendikulyar bo`lsa, k ni toping.

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Yechish. $\vec{a}\cdot\vec{b} = x_1x_2+y_1y_2 = 3\,k - 18 = 0$.

Bundan $k = 6$ ekanligi kelib chiqadi.

$\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko`paytmasi uchun quyidagi formula o`rinli:

$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot cos\alpha$,bunda $\alpha -$ $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak.

4-misol.Agar $|\vec{a}| = 2$, $|\vec{b}| = 3$ bo`lib,ular orasidagi burchak $60^0$ bo`lsa, $2\vec{a} - \vec{b}$ va $2\vec{a}+3\vec{b}$ vektorlarning skalyar ko`paytmasini toping.

Yechish.

$(2\vec{a} - \vec{b})\cdot(2\vec{a}+3\vec{b}) = 4\vec{a}^2 + 4\vec{a}\cdot\vec{b} - 3\vec{b}^2 = 4\cdot4 + 4\cdot|\vec{a}|\cdot|\vec{b}|\cdot cos60^0 - 3\cdot9 = 16 + 12 - 27 = 1.$

Demak, $2\vec{a} - \vec{b}$ va $2\vec{a}+3\vec{b}$ vektorlarning skalyar ko`paytmasi 1 ga teng ekan.

II.Vektorlarning vektor ko`paytmasi.

$\vec{a}$ va $\vec{b}$ vektorlarning vektor ko`paytmasi $\vec{a}\times\vec{b}$ ko`rinishida yoziladi va quyidagi formula bo`yicha hisoblanadi.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = i\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix} - j\begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix} + k\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$$

Bu yerda $\vec{i}(1;0;0)$, $\vec{j}(0;1;0)$, $\vec{k}(0;0;1)$ vektorlar mos ravishda koordinata o`qlari bo`yicha yo`nalgan birlik vektorlar.

5-misol. $\vec{a}(2;-1;3)$ va $\vec{b}(-2;4;1)$ vektorlarning vektor ko`paytmasini toping.

Yechish.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 3 \\ -2 & 4 & 1 \end{vmatrix} = i\begin{vmatrix} -1 & 3 \\ 4 & 1 \end{vmatrix} - j\begin{vmatrix} 2 & 3 \\ -2 & 1 \end{vmatrix} + k\begin{vmatrix} 2 & -1 \\ -2 & 4 \end{vmatrix} = -13i - 8j + 6k.$$

Vektorlarning vektor ko`paytmasi yana vektor kattalikdan iborat bo`ladi.

Ikkita vektorga qurilgan parallelogrammning yuzi shu vektorlarning vektor ko`paytmasidan iborat bo`lgan vektorning, absolyut qiymatiga teng bo`ladi, ya`ni

$S = |\vec{a}\times\vec{b}|$, bunda S parallelogrammning yuzi.

6-misol. $\vec{a}(1;2;-2)$ va $\vec{b}(2;-3;4)$ vektorlarga qurilgan uchburchakning yuzini toping.

Yechish.

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$\vec{a}\ \mathrm{X}\ \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = i\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix} - j\begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix} + k\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$ formuladan foydalanib berilgan vektorlarning vektor ko`paytmasini topamiz.

$$\vec{a}\ \mathrm{X}\ \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & -2 \\ 2 & -3 & 4 \end{vmatrix} = 14\vec{i} - 8\vec{j} - 7\vec{k}.$$

$\vec{a}\ \mathrm{X}\ \vec{b}$ vektorning koordinatasi (14;-8;-7) ga teng. Bu vektorning absolyut qiymati berilgan vektorlarga qurilgan parallelogramning yuziga teng bo`ladi.Uchburchakning yuzi esa parallelogramm yuzining yarmiga teng.

Demak, $\vec{a}(1;2;-2)$ va $\vec{b}(2;-3;4)$ vektorlarga qurilgan uchburchakning yuzi: $S = \frac{1}{2}\sqrt{14^2+8^2+7^2} = 0,5\sqrt{309}$.

III.Vektorlarning aralash ko`paytmasi.

$\vec{a}$ vektorning koordinatasi $(x_1; y_1; z_1)$,$\vec{b}$ vektorning koordinatasi $(x_2; y_2; z_2)$ va $\vec{c}$ vektorning koordinatasi $(x_3; y_3; z_3)$ bo`lsin.Bu vektorlarning aralash ko`paytmasini topish uchun, $\vec{a}$ va $\vec{b}$ vektorlarning vektor ko`paytmasidan iborat vektorni $\vec{c}$ vektorga skalyar ko`paytiramiz.

$\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlarning aralash ko`paytmasi $\vec{a}\vec{b}\vec{c}$ ko`rinishda yoziladi va quyidagi formula bo`yicha hisoblanadi.

$\vec{a}\vec{b}\vec{c} = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$ .Vektorlarning aralash ko`paytmasi son kattalikdan iborat.

7-misol. $\vec{a}(5;-2;2)$, $\vec{b}(4;2;-3)$ va $\vec{c}(4;-3;1)$ vektorlarning aralash ko`paytmasini toping.

Yechish.

$$\vec{a}\vec{b}\vec{c} = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = \begin{vmatrix} 5 & -2 & 2 \\ 4 & 2 & -3 \\ 4 & -3 & 1 \end{vmatrix} = 10 + 24 - 24 - 16 - 45 + 8 =- 43$$

Demak, $\vec{a}(5;-2;2)$, $\vec{b}(4;2;-3)$ va $\vec{c}(4;-3;1)$ vektorlarning aralash ko`paytmasi -43 ga teng ekan.

8-misol. $\vec{a}(3;2;1)$, $\vec{b}(5;1;-3)$ va $\vec{c}(2;3;2)$ vektorlarga qurilgan parallelepipedning hajmini toping.

Yechish.Vektorlarning aralash ko`paytmasi formulasiga ko`ra:

$$\vec{a}\vec{b}\vec{c} = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = \begin{vmatrix} 3 & 2 & 1 \\ 5 & 1 & -3 \\ 2 & 3 & 2 \end{vmatrix} = 6 - 12 + 15 - 2 + 27 - 20 = 14.$$

Demak, $\vec{a}(3;2;1)$, $\vec{b}(5;1;-3)$ va $\vec{c}(2;3;2)$ vektorlarga qurilgan parallelepipedning hajmi 14 ga teng.

Mustaqil yechish uchun masalalar·

1. $\vec{a}(5;-3;8)$ va $\vec{b}(3;-6;-1)$ vektorlarning skalyar ko`paytmasini toping.
2. $\vec{a}(1;-4;4)$ va $\vec{b}(6;-2;m)$ vektorlarning skalyar ko`paytmasi 47 ga teng bo`lsa,m ni toping.
3. $\vec{a}(4;6)$ va $\vec{b}(k;-8)$ vektorlar perpendikulyar bo`lsa, k ni toping.
4. $\vec{a}(3;-4;3)$ va $\vec{b}(-2;5;12)$ vektorlarning vektor ko`paytmasini toping.
5. $\vec{a}(7;-2;2)$ va $\vec{b}(6;-5;4)$ vektorlarga qurilgan uchburchakning yuzini toping.
6. $\vec{a}(1;-3;2)$, $\vec{b}(4;2;-1)$ va $\vec{c}(5;-2;1)$ vektorlarning aralash ko`paytmasini toping.
7. $\vec{a}(4;-2;8)$, $\vec{b}(-4;1;-3)$ va $\vec{c}(9;-3;2)$ vektorlarga qurilgan parallelepipedning hajmini toping.
8. $\vec{a}(5;-2;2)$, $\vec{b}(4;2;-3)$ va $\vec{c}(4;-3;1)$ vektorlarning aralash ko`paytmasini toping.

XULOSALAR

Biz yuqorida vektorlarning xossalari va vektorlardan foydalanib vektorlarning skalyar ko`paytmasi vektor ko`paytmasi va aralash ko`paytmasi va unga oid misollarni yechish metodlarini ko`rib chiqdik.

Tanishib chiqilgan masalalardan ko`rinib turibdiki,turli masalalarni vektorlar yordamida qulay va tushunarli usulda hal qilish mumkin ekan.

Tavsiya etilgan masalalarning yechimlari, o`quvchilarimizni yangi bilim bilan boyitishga,ularni mantiqiy fikrlashlarini,tafakkurlarini va matematik tasavvurlarini rivojlantirishga xizmat qiladi.

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