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THE ROLE OF DIFFERENTIAL EQUATIONS IN MODELING NATURAL PHENOMENA

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Abstract. This article analyzes the role and importance of differential equations in modeling natural phenomena. It is shown that various physical and biological processes, such as population growth, radioactive decay, and heat dissipation, when expressed as mathematical models, can be analyzed accurately and efficiently using differential equations. It also provides information on the types of differential equations, their solution methods, and areas of application in practice.

Keywords: differential equation, modeling, population growth, radioactive decay, heat dissipation, mathematical model, Cauchy problem, general solution, particular solution.

Differential equations are equations involving unknown functions, their derivatives of various orders, and arbitrary variables. In these equations, the unknown function is defined by i, and in the first two i depends on one arbitrary variable t, and in the next ones on the arbitrary variables x, t and x, u, z, respectively. The theory of differential equations began to develop at the end of the 17th century, simultaneously with the emergence of differential and integral calculus. Differential equations are of great importance in mathematics, especially in its applications. Investigation of various problems in physics, mechanics, economics, engineering, and other fields leads to the solution of differential equations.

Also, one of the aspects that distinguishes mathematical modeling from other research methods is that in this method a real object or process is described by mathematical expressions - formulas, equations, functions.

For example, to describe the state of a physical or technological system, differential equations are constructed based on the physical laws associated with it.

A deep analysis of such models is carried out using mathematical methods - solving equations, evaluating functions, numerical calculations, etc. In most cases, these equations are in the form of linear or nonlinear differential equations, the solutions of which show how the process develops over time. Sometimes, even without solving these equations exactly, it is possible to draw important theoretical conclusions about an object or process by analyzing them qualitatively. For example, by analyzing the state of the equation solutions in phase space and the stability points, it is possible to predict how the system will behave. Nowadays, there is almost no scientific or practical field that does not use mathematical modeling. Large-scale and deep results are achieved using this method in physics (especially theoretical physics), biology, chemistry, economics, ecology, and even in the social sciences. For example, in biology, the Lotka-Volterra equations, which express population growth, and in economics, the relationships between macroeconomic indicators are modeled using differential equations. [2; 18-23-b.]

Mathematical modeling also plays an important role in assessing the efficiency of various technological processes, in particular, production, energy or transport systems.

These systems are multi-parameter, and systems of differential equations are usually used to represent them. With the help of these systems, the interactions between parameters are determined.

It is necessary to separately note several advantages of mathematical modeling over other methods.

The first advantage is its economic efficiency. It is not necessary to build expensive laboratory or industrial devices to conduct experiments, which saves material resources.

By solving differential equations based on the model on a computer, it is possible to conduct numerical simulations that replace experiments.

Sometimes it is completely impossible to build such physical structures in real life (for example, explosions, space events).

Therefore, modeling is the safest and most effective way to study processes, especially those that occur under dangerous or uncertain conditions. The second advantage is the ability to predict future situations based on general patterns.

For example, the development of atmospheric, technological, or economic systems can be predicted using equations such as $\partial T/\partial t = f(x,t)$, which describe how temperature or pressure changes over time.

Another important aspect of mathematical modeling is the ability to analyze different options and determine optimal conditions by changing the parameters.

This allows you to evaluate how the system behaves under different conditions by changing the initial or boundary conditions of the differential equations. [3; 220-221-b.]

Differential equations with special derivatives An important feature of these equations, which differs from ordinary differential equations, is that the set of all their solutions, that is, the "general solution", does not depend on arbitrary constants, but on arbitrary functions; in general, the number of these arbitrary functions is equal to the order of the differential equation; and the number of their arbitrary variables is one less than the number of variables of the solution being sought. Solving a differential equation with a first-order partial derivative in one unknown leads to the solution of a system of ordinary differential equations. In the theory of differential equations with special derivatives of order higher than one, various boundary value problems are investigated in the same way as the Cauchy problem.

The term differential equation was introduced by Leibniz. His follower Bernoulli (1654-1705) used series expansion to solve differential equations. Bernoulli used the introduction of parameters $y = p$ to reduce the order of a second-order equation that does not contain one variable.

The subsequent development of the theory of differential equations was greatly contributed by Euler (1707-1783), an academician of the Petersburg Academy, and the French mathematicians Clairault (1713-1765), d'Alembert (1717-1783), and Lagrange (1736-1813). Euler introduced the substitution $y = e\ kx$ to solve a linear homogeneous equation.

D'Alembert showed that the general solution of a linear nonlinear equation is equal to the sum of its particular solutions and the general solution of the corresponding linear homogeneous equation. Lagrange developed the method of variation of constants. Lagrange proved that if r particular solutions of a linear homogeneous equation are known, it can be reduced to r of order. Euler and

Clairault proposed the introduction of an integrating multiplier and introduced the condition for the integration of the equation M dx + N dx = 0. Particular solutions were first encountered by B. Taylor (1685-1731) in 1715. In 1736, A. Clairault distinguished between particular and general solutions.

Many natural phenomena vary over time and space. For example, processes such as rising sea levels, changes in atmospheric temperature, population growth or decline, and changes in chemical concentrations are among such phenomena. Mathematical models are used to analyze, understand, and predict these processes. Differential equations play a key role in these models.

Population growth is a natural process that plays an important role in demography. One of the simplest and most popular models for modeling this process is the exponential growth model:

$dP/dt = rP$

According to this model, the population grows at a constant rate. But in practice, this trend is not always correct, because natural resources, feed, space and other factors are limited. Therefore, a logistic model is used to better represent population growth:

$dP/dt = rP(1 - P/K)$

Radioactive decay is a nuclear process that is also modeled by a differential equation.

His model:

$dN/dt = -\lambda N$

This equation represents exponential decay, and the solution is:

$N(t) = N_0 e^{(-\lambda t)}$

This model is used in nuclear physics, archaeology, medicine and environmental analysis.

Heat dissipation is an important process in physics, which is also modeled by differential equations.

Fourier's equation:

$\partial u/\partial t = \alpha\, \partial^2 u/\partial x^2$

This model is widely used in fields such as construction, energy, geophysics, and medicine.

Problem 1: Population growth (exponential model) Condition: The population of a city was 100,000 in 2020. If the population grows by 2% per year, find the population after 10 years.

Solution: Exponential growth model: $dP/dt = rP$, $r = 0.02$

Solution: $P(t) = P_0 e^{rt} = 100000 \cdot e^{0.02 \cdot 10} \approx 100000 \cdot 1.2214 = 122140$

Answer: After 10 years, the population will be approximately 122,140 people.

Problem 2: Radioactive decay

Condition: The initial amount of radioactive material is 80 g. If its decay constant is $\lambda = 0.1$, how much material will remain after 5 hours?

Solution:

Model: $dN/dt = -\lambda N$, $\lambda = 0.1$

Solution: $N(t) = N_0 e^{-\lambda t} = 80 \cdot e^{-0.1 \cdot 5} \approx 80 \cdot 0.6065 = 48.52$ g

Answer: Approximately 48.52 g of material will remain after 5 hours.

Problem 3: Heat dissipation (simple case)

Condition: The temperature at the end of a long iron rod is $u(0,t) = 100°C$, and at other points $u(x,0) = 0°C$. The heat dissipation equation is $\partial u/\partial t = \alpha\, \partial^2 u/\partial x^2$. If $\alpha = 0.01$, $x = 1$, $t = 10$, find the approximate temperature.

Solution (approximate):

Solution: $u(x,t) \approx (4/\pi) \sum (1/(2n+1)) \cdot \sin((2n+1)\pi x) \cdot e^{-(2n+1)^2 \pi^2 \alpha t}$

1st approximation: $u(1,10) \approx 0$ (because $\sin(\pi) = 0$)

Answer: The temperature will be very low; approximately close to 0°C.

Differential equations are also used to model processes such as motion in mechanics, dynamics of chemical reactions, and metabolism in ecology and biology.

For example, Newton's second law:

$m\, d^2x/dt^2 = F(x,t)$,

reaction rate equation:

$d[A]/dt = -k[A][B]$,

and population models (Lotka-Volterra).

Analytical, numerical, and symbolic methods are used to solve differential equations. Today, computer technology has made this process even easier (for example, MATLAB, Mathematica, Python).

It is also worth noting that models of atmospheric motion can be constructed using differential equations based on Newton's laws of motion or the second law of thermodynamics.

Since changes in the atmosphere are dynamic in time and space, multivariable differential equations, such as the Navier-Stokes equations or heat transfer equations for energy exchange, are used to describe them.

Differential equations are an important tool in modeling atmospheric phenomena. For example, differential equations of various orders are used to describe how meteorological quantities such as temperature, pressure, humidity, or wind speed change in time and space.

An example of this is the equation $\partial T/\partial t = k \nabla^2 T$ (heat transfer), which describes the change in air temperature $T(x,t)$ over time.

Each field has its own model, which is mathematically expressed in terms of differential equations.

The Navier-Stokes equations (higher-order differential equations) play a key role in describing atmospheric motion. Turbulence is modeled by nonlinear Navier-Stokes equations.

At the same time, this field of science is aimed at developing methods for predicting atmospheric processes through numerical (hydrodynamic) modeling. [5; 44-57-b.]

In this case, differential equations are solved using numerical methods - for example, numerical integration methods (Euler, Runge-Kutta).In conclusion, differential equations are an important tool in understanding and modeling natural phenomena. With their help, processes can be analyzed, controlled and predicted. Their role in modern science and technology is incomparable.

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