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INEQUALITIES OF PROOF DIFFERENT METHODS

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Annotation: This in the article In mathematics numerical inequalities to prove circle very much many issues is found . Many inequalities in proof inequalities properties and fact from the beginning known was from inequality is used . Initially numerical inequalities properties with let's get acquainted :

Key words: Equality sign , inequality , middle geometric , medium arithmetic.

1. Definition: If $a-b$ the difference positive number if , a number b from the number big it is said and this attitude $a>b$ in the form of written . If $a-b$ the difference negative number if , a number b from the number small it is said and this attitude $a<b$ in the form of written .

Any a and b numbers for following three from relationships only one appropriate :

1. $a-b>0 \quad a>b$
2. $a-b<0 \quad a<b$
3. $a-b=0 \quad a=b$.

Numerical inequalities following to properties has :
1^0. If $a>b$ and $b>c$ if , $a>c$ will be .
2^0. If $a>b$ and $c>0$ if , $a\ \ c>b\ \ c$ will be .
3^0. If $a+b>c$ if , $a>c-b$ will be .
4^0. If $a>b$ and $c\ \ R$ if , $a+c>b+c$ will be .
4^0. If $a>b$ and $c<0$ if , $a\ \ c<b\ \ c$ will be .
5^0. If $a>b$ and $c>d$ if , $a+c>b+d$ will be .
6^0. If $a>b>0$ and $c>d>0$ if , $a\ \ c>b\ \ d$ will be .
7^0. If $a>b\ \ 0$ and $n\ \ N$ if , $a^n>b^n$ will be (if n – it is an odd number , $b>0$ condition excess is considered)
8^0. If $a\ \ b>0$ and $c>d>0$ if , $\frac{a}{d}>\frac{b}{c}$ will be .
9^0. If $a>b$ and $n\ \ N$ if , $a^{2n+1}>b^{2n+1}$ will be .
9^0. If $a>b\ \ 0$ and $n\ \ N$ if , $\sqrt[n]{a}>\sqrt[n]{b}$ will be .
10^0. If $a>b$ and $n\ \ N$ If it is , $\sqrt[2n+1]{a}>\sqrt[2n+1]{b}$ it will be .

Inequalities prove

I. Inequalities from the definition using prove

To the definition according to $a>b$ to be for $a-b$ to be a positive number need . That's why for given $a,b,...,k$ values in the collection $f(a,b,...,k)>g(a,b,...,k)$ inequality proof for $f(a,b,...,k)-g(a,b,...,k)$ difference we build and he $a,b,...,k$ of given in values positive to the fact that confidence harvest we do . Just like also this from the method $f<g, f\ \ g, f\ \ g$ inequalities is also used in proof .

Example 1. $a\ \ 0, b\ \ 0$ if , $\frac{a+b}{2}\ \ \sqrt{ab}$ (Cauchy inequality) prove .

Proof . $\frac{a+b}{2}-\sqrt{ab}$ difference we build and his/her gesture Let's find out .

$$\frac{a+b}{2}-\sqrt{ab}=\frac{a-2\sqrt{ab}+b}{2}=\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{2}$$

$\frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{2}$ expression optional negative not been a and b in values negative will not be . Hence , $\frac{a+b}{2} \quad \sqrt{ab}$. Equality sign It is $a=b$ also appropriate .

Example 2. If $ab>0$ if , $\frac{a}{b}+\frac{b}{a} \quad 2$ to be prove .

Proof . $\frac{a}{b}+\frac{b}{a} \quad -2=\frac{a^2+b^2-2ab}{ab}=\frac{(a-b)^2}{ab}$ $ab>0$ that was for $\frac{(a-b)^2}{ab} \quad 0$. Equality sign is $a=b$ also appropriate . So , $\frac{a}{b}+\frac{b}{a} \quad -2$ difference negative Not . It has been proven .

Example 3. This $a^2+4b^2+3c^2+14>2a+12b+6c$ inequality prove .

Proof . $\left(a^2+4b^2+3c^2+14\right)-\left(2a+12b+6c\right)$ difference We build it . Its terms we group .

$$\left(a^2-2a+1\right)+\left(4b^2-12b+9\right)+\left(3c^2-6c+3\right)+1=(a-1)^2+(2b-3)^2+3(c-1)^2+1.$$

Last expression optional It is a,b,c also valid . It has been proven .

Example 4. If $a+b+c \quad 0$ if , $a^3+b^3+c^3 \quad 3abc$ to be prove .

` Proof . $a^3+b^3+c^3-3abc$ difference Let's see . a^3+b^3 expression of the sum to the cube we fill .

$$a^3+b^3+c^3-3abc=a^3+3a^2b+3ab^2+b^3+c^3-3a^2b-3ab^2-3abc=(a+b)^3-3ab(a+b+c)+c^3$$

Now $(a+b)^3+c^3$ what to multipliers we separate .

$$(a+b)^3+c^3-3ab(a+b+c)=((a+b)+c)\left((a+b)^2-(a+b)c+c^2\right)-3ab(a+b+c)=$$

$$=(a+b+c)\left(a^2+2ab+b^2-ac-bc+c^2-3ab\right)=(a+b+c)\left(a^2+b^2+c^2-ab-bc-ac\right)=$$

$$=\frac{1}{2}(a+b+c)\left(2a^2+2b^2+2c^2-2ab-2bc-2ac\right)=\frac{1}{2}(a+b+c)\left((a-b)^2+(a-c)^2+(b-c)^2\right)$$

On condition according to $a+b+c \quad 0$. Second bracket negative inside not . So , the expression negative not . In inequality equality sign $a+b+c=0$ or $a=b=c$ appropriate will be .

II. Inequalities of proof artificial methods .

This method essence as follows :

Proving inequality one row form from substitutions after from certain (base) inequalities to one is brought . Base inequalities as for example following from inequalities is used .

a) $a^2 \quad 0$ b) $\frac{a+b}{2} \quad \sqrt{ab}$, in which $a \quad 0, b \quad 0$ d) $\frac{a}{b}+\frac{b}{a} \quad 0$, in which $ab>0$

e) $ax^2+bx+c>0$, in which $a>0, b^2-4ac<0$.

Example 5. If $a \quad 0, b \quad 0, c \quad 0, d \quad 0$ if , $\frac{a+b+c+d}{4} \quad \sqrt[4]{abcd}$ that prove .

Proof . Basis inequality as basket inequality we will get

$$\frac{\frac{a+b}{2}+\frac{c+d}{2}}{2} \quad \sqrt{\frac{a+b}{2}\;\frac{c+d}{2}}.$$

Own in turn $\frac{a+b}{2} \quad \sqrt{ab}$ and $\frac{c+d}{2} \quad \sqrt{cd}$ that was for

$$\sqrt{\frac{a+b}{2}\;\frac{c+d}{2}} \quad \sqrt{\sqrt{ab}\;\sqrt{cd}}=\sqrt[4]{abcd}.$$

So , $\frac{\frac{a+b}{2}+\frac{c+d}{2}}{2} \quad \sqrt[4]{abcd}$. But $\frac{\frac{a+b}{2}+\frac{c+d}{2}}{2}=\frac{a+b+c+d}{4}$.

So as $\frac{a+b+c+d}{4} \quad \sqrt[4]{abcd}$ Proof analysis as following to the conclusion Let's go . In inequality equality sign $a=b, c=d$ and $\frac{a+b}{2}=\frac{c+d}{2}$ when appropriate .

Example 6. This $\left(\frac{n+1}{2}\right)^n > n!$, in this $n \quad N, n>1$ inequality prove .

Proof.Base inequalities as following inequalities we will get

$$\frac{n+1}{2} \quad \sqrt{n\;1};\frac{(n-1)+2}{2} \quad \sqrt{(n-1)\;2};\ \frac{(n-2)+3}{2} \quad \sqrt{(n-2)\;3};...;$$

$$\frac{2+(n-1)}{2} \quad \sqrt{2\;(n+1)};\ \frac{1+n}{2} \quad \sqrt{1\;n}.$$

These n inequalities multiply ,

$$\left(\frac{n+1}{2}\right)^n \quad \sqrt{(n(n-1)(n-2)\;...\;2\;1)(1\;2\;3\;...\;(n-1)n)}=\sqrt{n!\;n!}=\sqrt{(n!)^2}=n!$$

what harvest We do . So , $\left(\frac{n+1}{2}\right)^n \quad n!$.

On condition according to $n \quad 1$ that was for Koshi's support from inequalities the first only strict to be possible . In that case support inequalities multiplication as a result harvest was last inequality strict It will be . So so ,

$$\left(\frac{n+1}{2}\right)^n > n!.$$

Example 7. If $a>0, b>0, c>0$ if , $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \quad 9$ that prove .

Proof . Basis inequalities as following inequalities we will get

$$\frac{a}{b}+\frac{b}{a} \quad 2;\qquad \frac{a}{c}+\frac{c}{a} \quad 2;\qquad \frac{b}{c}+\frac{c}{b} \quad 2.$$

These inequalities $a=b, a=c$ and $b=c$ when appropriate will be . They adding

$$\frac{a}{b}+\frac{b}{a}+\frac{a}{c}+\frac{c}{a}+\frac{b}{c}+\frac{c}{b} \quad 6$$

or

$$\frac{b+c}{a}+\frac{a+c}{b}+\frac{a+b}{c} \quad 6$$

what harvest we will do

$$1+\frac{a+c}{b} \quad + \quad 1+\frac{b+c}{a} \quad + \quad 1+\frac{a+b}{c} \qquad 9,$$

$$\frac{a+b+c}{b}+\frac{a+b+c}{a}+\frac{a+b+c}{c} \quad 9.$$

Equality sign It is $a=b=c$ also appropriate .

Example 8. If $n \quad N, n>1$ if , $\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+...+\frac{1}{n^2}<1$ to be prove .

Proof . $\frac{1}{4}=\frac{1}{2\,2}<\frac{1}{1\,2}$; $\quad \frac{1}{9}=\frac{1}{3\,3}<\frac{1}{2\,3}$; $\quad \frac{1}{16}=\frac{1}{4\,4}<\frac{1}{3\,4}$;...; $\frac{1}{n^2}=\frac{1}{n\,n}<\frac{1}{(n-1)n}$.

These $(n-1)$ inequalities adding ,

$$\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+...+\frac{1}{n^2}<\frac{1}{1\,2}+\frac{1}{2\,3}+...+\frac{1}{(n-1)n}=\frac{2-1}{1\,2}+\frac{3-2}{2\,3}+\frac{4-3}{3\,4}+...+\frac{n-(n-1)}{(n-1)n}=$$

$$= \quad 1-\frac{1}{2} \quad + \quad \frac{1}{2}-\frac{1}{3} \quad + \quad \frac{1}{3}-\frac{1}{4} \quad +...+ \quad \frac{1}{n-1}-\frac{1}{n} \quad =1-\frac{1}{n}<1$$

what harvest We do . So , $\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+...+\frac{1}{n^2}<1$.

III. Inversely hypothesis to do method using inequalities prove

Example 9. If $a \quad 0, b \quad 0, c \quad 0, d \quad 0$ if , $\sqrt{(a+c)(b+d)} \quad \sqrt{ab}+\sqrt{cd}$ that prove .

Proof . Optional . negative not been a,b,c,d numbers for given inequality appropriate that our proof need . Conversely hypothesis We do . Negative not been a,b,c,d values for $\sqrt{(a+c)(b+d)}<\sqrt{ab}+\sqrt{cd}$ inequality appropriate Let this inequality be every two part negative absence for his/her every two part square lift ,

$$(a+c)(b+d)<ab+cd+2\sqrt{abcd}$$

inequality harvest we will do . From now on

$$bc+ad<2\sqrt{abcd}\ ;\ \frac{bc+ad}{2}<\sqrt{(bc)(ad)}.$$

But this Koshi to inequality is contradictory . So , our our hypothesis wrong . Therefore for given inequality That's right .

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