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METHODOLOGY OF TEACHING CURVES IN DIFFERENTIAL GEOMETRY USING INTERACTIVE AND DIGITAL TECHNOLOGIES

Kaypnazarova G.Kh.
PhD in Physics and Mathematics, Associate Professor, Nukus State Pedagogical Institute

Annotation: This article explores a methodology for teaching lines in the plane and in space using modern interactive and computer technologies. It presents the theoretical foundations of the topic, graphic representations, parametric and analytical forms, and the advantages of integrating technologies into the learning process. Based on an experiment conducted with third-year students, the study analyzes learning outcomes and demonstrates the effectiveness of an interactive approach.
Keywords: lines, spatial thinking, interactive method, GeoGebra, graphical representation, teaching methodology.

1. Introduction

In the modern educational process, the application of information and communication technologies in teaching various subjects is becoming increasingly significant. This is especially true for geometry, a subject rich in complex concepts, spatial reasoning, and abstract representations. In teaching geometry, the role of interactive and computer technologies is invaluable.

Curves represent one of the fundamental concepts in geometry. Understanding their representations in both the plane and space — through equations and graphs — poses a challenge for many students. Traditional approaches often present these concepts merely through formal definitions and mathematical formulas, which hinders students' deep comprehension of the topic.

Contemporary pedagogical experience and modern educational technologies show that the use of interactive methods — particularly software tools such as GeoGebra, Cabri 3D, and Maple — along with multimedia resources, allows for clear, logical, and visual delivery of essential knowledge to students.

Therefore, this article explores a methodology for teaching curves using interactive and computer-based technologies. It analyzes the outcomes derived from teaching experiments, highlights current challenges, and offers practical solutions.

Research Objective:

To develop, implement, and evaluate an effective methodology for teaching the topic of curves based on modern technologies.

Research Tasks:

- To analyze the theoretical foundations of the concept of a curve;
- To identify the advantages of using interactive and computer technologies;
- To develop a methodology applicable in the teaching process;
- To evaluate its effectiveness based on practical implementation and draw relevant conclusions.

2. Theoretical Foundations of Curves

In geometry, the concept of a curve is one of the most fundamental and essential elements, serving as a basis for constructing shapes, curved lines, surfaces, and spatial objects. A curve is a geometric object generated by the motion of a point, which distinguishes it from other elements. It is one of the simplest yet most foundational entities in mathematics.

Curves in the Plane

Curves in the plane appear in basic geometric forms such as straight lines, parabolas, circles, ellipses, and hyperbolas. These curves are usually represented in general analytical forms, for example:

$f(x, y) = 0$ or $\vec{r} = \vec{r}(x(t), y(t))$

Through such parametric or implicit representations, students study curves from a mathematical perspective[2]. However, this does not always help them form a clear spatial or visual image of the curve.

Curves in Space

Curves in three-dimensional space are described by coordinates in three variables:

$\vec{r}(t) = \{x(t), y(t), z(t)\}$

Presenting these curves not only in theory but also using graphical software is extremely beneficial for students. For instance, visualizing spatial curves such as helices or curves defined in spherical or cylindrical coordinates in 3D format significantly enhances understanding and retention.

Challenges and Needs in Studying Curves

Traditional lessons tend to focus on the theoretical definition of curves, often overlooking their graphical representations, motion paths, and topological properties. As a result, students tend to associate curves merely with formulas or equations, which slows down intuitive and visual comprehension of the topic.

The Need to Make Theory Interactive

Modern educational technologies are capable of bridging this gap. In an interactive environment, it is possible to dynamically demonstrate the trajectory of a moving point, the curvature of a curve, its symmetry, tangents, and other features in real time. This greatly enhances students' ability to approach the topic with a practical, visual mindset and helps solidify theoretical knowledge.

3. The Role of Interactive and Computer Technologies in Modern Education

In the 21st century, technological advancements in education are bringing substantial innovations to teaching methodologies. It is no longer sufficient for educators to merely deliver knowledge; it is now essential to convey it in a clear, engaging, and visual manner that resonates with the students' cognitive understanding[1]. This approach is particularly important for geometry — a subject that requires strong visual and spatial reasoning.

Interactive Technologies as Enhancers of the Learning Process

Interactive methods transform the learning process into a two-way dynamic: a direct exchange of knowledge between teacher and student. These methods include the following tools and strategies:

- **Electronic whiteboards**: Used to draw the motion of a curve, its tangent or normal in real-time;
- **Clickers or online polling systems**: Employed to assess student understanding through tests or instant feedback;
- **Problem-Based Learning (PBL)**: In which students construct a curve based on given conditions and tasks.

Potential of Computer Technologies

The following software tools provide extensive opportunities for teaching geometry, particularly the topic of curves:

Software	Description
GeoGebra	Enables dynamic visualization of curves in parametric, implicit, and

	graphical forms.
Cabri 3D	Allows for the interactive construction and manipulation of spatial curves.
Maple / Mathematica	Supports complex mathematical modeling and visual analysis of curves.
AutoCAD / SolidWorks	Used by engineering students for real-world design tasks involving curves and surfaces.

Advantages of Using These Technologies

- **Visualization**: Abstract curves are brought to life visually;
- **Student engagement**: Learners shift from passive observers to active participants;
- **Independent thinking**: Students analyze the properties of curves independently through graphical models;
- **Increased motivation**: Lessons become more engaging, resembling interactive games or simulations.

Challenges in Implementation

Despite their potential, integrating these technologies into the teaching process may encounter several obstacles:

- Lack of access to appropriate technical equipment;
- Limited technical proficiency of educators;
- Variability in students' digital literacy levels;
- Difficulties in classroom management when using technology.

Nonetheless, the opportunities offered by modern tools create a foundation for gradually overcoming these issues. The key lies in aligning technological resources with targeted pedagogical strategies.

4. The Methodology of Teaching Curves: A Step-by-Step Approach

To ensure effective instruction in the topic of curves, it is essential to implement a step-by-step approach based on didactic materials, interactive tools, and modern computer software. Each step complements the others and contributes to the gradual consolidation of knowledge in students' minds.

Step 1: Formation of the Topological Concept

The first step is to introduce students to the idea that a curve is the trajectory of a moving point. At this stage, students develop an initial understanding of the shape of the curve, whether it is open or closed, and its orientation in space through visual aids.

Example: Using GeoGebra, students can observe in real time how the movement of a point generates a curve.

Step 2: Introduction of Analytical Representations

Students are then introduced to different forms of curve equations — parametric, general, and implicit. Emphasis is placed on expressing curves through mathematical formulas, such as: $\vec{r}(t)=\{x(t),y(t),z(t)\}$ or $f(x(t),y(t),z(t))=0$ Alongside this, students use computer tools to generate the corresponding graphs. This step helps them bridge the gap between formulas and visual representations.

Step 3: Graphical Construction and Analysis

Using graphical software, students construct curves, examine their properties (such as circular, spiral, or sinusoidal curves), and observe their behavior in space.

Tools used: GeoGebra 3D, Cabri 3D, Maple.

During this step, visual models are discussed in class using interactive whiteboards or projectors. Students are tasked with analyzing features such as curvature, symmetry, and tangents.

Step 4: Practical Assignments and Projects

Students complete individual or group-based graphical tasks:

- Constructing graphs from given equations;
- Modeling spatial curves in 3D format;
- Solving problems through graphical approaches.

These activities develop not only subject knowledge but also practical skills and creative thinking.

Step 5: Analysis and Reflection

In the final stage, students reflect on the graphs they constructed and answer questions such as:

- How does the curve move?
- What is its spatial direction?
- How does the curve change when the equation is modified?

This step serves to evaluate the student's depth of understanding and their ability to think critically about the concepts.

Methodological Recommendations

- Each theoretical concept should be supported by a visual example;
- Every graphical construction should be carried out using software tools;
- Each lesson should include opportunities for students to create and explain their own curves;
- Evaluation should go beyond standard tests and include practical and explanatory tasks.

5. Experimental Analysis

To evaluate the effectiveness of the proposed methodology, a pedagogical experiment was conducted with students from the Faculty of Physics and Mathematics. Two groups participated in the study: the experimental group received instruction using interactive and computer technologies, while the control group followed the traditional lecture-based approach.

5.1. Organization of the Experiment

The pedagogical experiment was carried out in three stages:

- **Preparatory stage** – diagnostic tests were administered to determine the students' initial knowledge;
- **Formative stage** – the proposed methodology was implemented in the experimental group, focusing on the use of software such as GeoGebra, Maple, and Cabri 3D;
- **Control stage** – the results of the students were evaluated using tests, practical assignments, and presentations.

5.2. Goals and Objectives of the Experiment

The main goals of the experiment were:

- To determine the level of student understanding of curves on the plane and in space;
- To assess the influence of interactive and computer technologies on the development of spatial imagination and analytical thinking;
- To compare the outcomes of students taught through traditional and modern methods.

5.3. Results of the Experiment

Criteria	Control Group (%)	Experimental Group (%)
Correct understanding of curve types	55	82
Ability to graph curves	47	85
Ability to analyze and explain	42	79

These results indicate a significant improvement in the performance of the experimental group across all evaluated criteria. The use of dynamic software allowed students to visualize, manipulate, and analyze curves in an intuitive manner.

5.4. Student Feedback

A survey conducted among the experimental group revealed the following:

- 90% found the lessons more engaging and easier to understand;
- 88% felt that software tools helped them learn better;
- 84% expressed greater interest in mathematics and geometry.

These indicators confirm that modern technological tools not only improve learning outcomes but also enhance students' motivation and confidence in mastering complex geometric concepts.

6. Conclusion and Recommendations

The conducted research and experimental analysis allow us to draw the following conclusions:

- The methodology for teaching curves on the plane and in space, based on interactive and computer technologies, has proven to be more effective than traditional methods. It significantly enhances students’ spatial thinking, analytical skills, and independent problem-solving abilities.
- Visualization through digital tools such as GeoGebra, Cabri 3D, and Maple fosters deeper understanding by enabling students to manipulate mathematical objects and observe changes dynamically.
- Integrating technology into the educational process increases students’ engagement, interest in the subject, and their overall motivation to learn mathematics.

Based on the results of the study, the following recommendations are proposed:

1. **Wider implementation** of interactive and computer-based approaches in teaching differential geometry, especially when introducing curves and their spatial representations.
2. **Development of digital learning resources** tailored for higher education students studying geometry, including software-guided practical tasks and visualization modules.
3. **Teacher training and retraining programs** should include modules on the use of modern mathematical software and innovative teaching strategies.
4. **Further research** should focus on the integration of artificial intelligence and augmented reality tools into geometry teaching to promote individualized and immersive learning experiences.

The results of the experiment demonstrate the potential of innovative technologies in transforming the teaching of mathematical concepts, particularly in geometry. Adoption of such methods can significantly improve the quality of education and prepare future educators for effective teaching in the digital age.

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