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THE MELLIN INTEGRAL TRANSFORM AND ITS APPLICATIONS

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Abstract

This article is devoted to the study of the Mellin integral transform and its applications in modern mathematical analysis. The paper examines the theoretical foundations of the Mellin transform, its fundamental properties, and its relationship with other classical integral transforms. Particular attention is paid to the role of the Mellin transform in solving problems involving scale invariance, asymptotic behavior, and multiplicative convolution. The study demonstrates that the Mellin transform provides an efficient analytical framework for evaluating improper integrals, analyzing special functions, and simplifying complex mathematical expressions. In addition, the article highlights practical applications of the Mellin transform in mathematical physics, signal and image processing, algorithm analysis, and other applied fields. The results confirm that the Mellin integral transform is a versatile and powerful tool that bridges theoretical and applied mathematics, offering broad potential for further research and development.

Keywords

Mellin integral transform; integral transforms; asymptotic analysis; special functions; scale invariance; mathematical analysis; applied mathematics

Introduction

The Mellin integral transform is a powerful mathematical tool that plays an important role in the analysis of functions, integral equations, and differential equations. It is particularly effective in problems involving scale invariance, asymptotic behavior, and multiplicative convolution. Due to its close relationship with other integral transforms, such as the Fourier and Laplace transforms, the Mellin transform occupies a significant position in modern mathematical analysis and applied mathematics [1].

Formally, the Mellin integral transform converts a given function into a complex function defined on the complex plane, allowing complex analytical methods to be applied to real-variable problems. This transformation has proven to be especially useful in number theory, probability theory, mathematical physics, and signal processing. Its ability to simplify complicated integral expressions and convert differential equations into algebraic forms makes it a valuable analytical technique [2].

In recent decades, the Mellin transform has gained increased attention due to its applications in asymptotic analysis and the study of special functions. Many classical functions, including the Gamma and Beta functions, can be naturally represented and analyzed using the Mellin transform framework. Moreover, the transform is widely used in the analysis of algorithms, fractal geometry, and the solution of boundary value problems arising in physics and engineering [3].

The interdisciplinary nature of the Mellin integral transform has led to its application in diverse scientific fields. In mathematical physics, it is applied to problems involving wave propagation and quantum mechanics, while in engineering it is used for signal and image analysis, particularly in scale-invariant pattern recognition. These applications demonstrate the versatility and practical importance of the Mellin transform in both theoretical and applied research [4].

This article aims to examine the Mellin integral transform and its main applications from a theoretical and applied perspective. The study is structured according to the IMRAD format and focuses on the fundamental properties of the Mellin transform, its methodological significance, and its role in solving mathematical and physical problems. By providing a systematic overview, the article seeks to highlight the relevance of the Mellin integral transform in contemporary mathematical research and its potential for further applications.

Materials and Methods

The materials of this study are based on classical and modern mathematical sources devoted to integral transforms, functional analysis, and applied mathematics, with particular emphasis on the Mellin integral transform. The primary sources include monographs, peer-reviewed journal articles, and textbooks that present the theoretical foundations, properties, and applications of the Mellin transform in various scientific domains. Special attention was given to works that discuss the relationship between the Mellin transform and other integral transforms, such as the Fourier and Laplace transforms, as well as its role in asymptotic analysis and special function theory [1].

The methodological framework of the research relies on an analytical and theoretical approach. The Mellin integral transform is examined through its formal definition, domain of convergence, and inversion formula. Standard methods of mathematical analysis, including complex analysis and functional analysis, are employed to study the transform's fundamental properties, such as linearity, scaling behavior, and convolution theorems. These methods allow the Mellin transform to be treated as a rigorous analytical tool applicable to a wide class of functions [2].

In addition, the study applies a comparative methodological approach to highlight similarities and differences between the Mellin transform and other widely used integral transforms. By comparing their operational properties and areas of applicability, the research identifies situations in which the Mellin transform provides advantages, particularly in problems involving multiplicative structures and scale invariance. This comparative analysis contributes to a deeper understanding of the methodological significance of the Mellin transform in applied mathematics [3].

To investigate practical applications, selected mathematical and physical problems from the literature were analyzed using the Mellin transform framework. These examples include the evaluation of improper integrals, asymptotic expansions of functions, and the analysis of special functions such as the Gamma function. The results obtained through Mellin-based techniques were interpreted using logical reasoning and synthesis, demonstrating the effectiveness of the transform in simplifying complex analytical problems [4].

Overall, the combination of theoretical analysis, comparative methods, and application-based examination provides a comprehensive methodological basis for studying the Mellin integral transform and its applications. This approach ensures both mathematical rigor and practical relevance, supporting the objectives of the present research.

Results

The analysis conducted in this study demonstrates that the Mellin integral transform provides a unified and efficient framework for solving a wide range of theoretical and applied problems. The results confirm that the Mellin transform is particularly effective in situations involving scale-invariant structures, asymptotic behavior, and multiplicative convolution. Compared to other classical integral transforms, it offers distinct analytical advantages in both mathematical and applied contexts [1].

One of the key results concerns the ability of the Mellin transform to simplify the evaluation of improper integrals and asymptotic expansions. By converting integral expressions into algebraic forms in the complex plane, the Mellin transform allows residues and contour integration techniques to be applied. This significantly reduces computational complexity and provides closed-form representations for many integrals that are otherwise difficult to evaluate using elementary methods [2].

Another important result is related to the application of the Mellin transform in the analysis of special functions. The study confirms that functions such as the Gamma and Beta functions naturally arise within the Mellin transform framework. This property enables a systematic derivation of functional equations, recurrence relations, and asymptotic estimates, which are essential in mathematical physics and analytic number theory [3].

The comparative analysis also reveals that the Mellin transform is more suitable than the Fourier and Laplace transforms for problems involving multiplicative scaling rather than additive structures. While Fourier and Laplace transforms are effective in time- or frequency-domain analysis, the Mellin transform excels in handling scale changes and power-law behaviors, which frequently occur in fractal analysis, algorithmic complexity, and signal processing [4].

In applied mathematics and engineering, the Mellin transform has shown strong potential in signal and image analysis, particularly in scale-invariant pattern recognition. The results indicate that Mellin-based techniques allow signals with different scales to be analyzed within a unified representation, improving robustness against scaling distortions [5]. This makes the Mellin transform a valuable tool in modern data analysis and computational applications.

The main application domains and analytical advantages of the Mellin transform identified in this study are summarized in Table 1.

Table 1

Applications of the Mellin Integral Transform

Application area	**Type of problem**	**Role of the Mellin transform**	**Analytical outcome**
Asymptotic	Improper integrals,	Conversion to algebraic	Simplified

Application area	**Type of problem**	**Role of the Mellin transform**	**Analytical outcome**
analysis	expansions	forms	evaluation
Special functions	Gamma, Beta functions	Functional representation	Closed-form relations
Mathematical physics	Boundary value problems	Integral equation solving	Analytical solutions
Signal processing	Scale-invariant signals	Scale normalization	Robust signal analysis
Algorithm analysis	Complexity estimation	Power-law modeling	Asymptotic bounds

In addition, the study identified specific mathematical properties of the Mellin transform that contribute to its wide applicability. These properties are summarized in Table 2.

Table 2

Key Properties of the Mellin Transform and Their Implications

Property	**Mathematical description**	**Practical implication**
Linearity	Transform of a sum equals sum of transforms	Simplifies complex expressions
Scaling property	Scale changes map to shifts in transform domain	Effective for scale-invariant problems
Convolution theorem	Multiplicative convolution becomes multiplication	Efficient problem-solving
Inversion formula	Original function can be recovered	Ensures analytical completeness

Overall, the results demonstrate that the Mellin integral transform is a versatile and powerful analytical tool with broad applicability across pure and applied mathematics. The findings support its growing relevance in modern research and confirm its advantages over other integral transforms in specific problem classes [6–8]. These results provide a strong foundation for further investigation into advanced applications of the Mellin transform in contemporary mathematical and engineering problems.

Discussion

The results obtained in this study confirm that the Mellin integral transform occupies a distinctive position among classical integral transforms due to its ability to effectively handle problems involving scale invariance and multiplicative structures. This observation is consistent with earlier theoretical studies, which emphasize the Mellin transform as a natural analytical tool for functions defined on the positive real axis and for problems exhibiting power-law behavior [1]. The findings further support the view that the Mellin transform complements, rather than replaces, the Fourier and Laplace transforms.

A key point of discussion concerns the efficiency of the Mellin transform in asymptotic analysis. The results demonstrate that many improper integrals and asymptotic expansions can be significantly simplified by transferring the problem into the complex domain and applying contour integration techniques. This approach aligns with classical methods in complex analysis and has been widely acknowledged as one of the main strengths of the Mellin transform [2]. Compared to direct real-variable methods, the Mellin-based approach provides clearer structural insights into the behavior of functions at infinity or near singularities.

The discussion of special functions highlights another important aspect of the Mellin transform. The natural emergence of the Gamma and Beta functions within the Mellin framework confirms its deep connection with analytic number theory and mathematical physics. Previous research has shown that many functional identities and recurrence relations of special functions can be systematically derived using Mellin techniques [3]. The present results reinforce this perspective and demonstrate the transform's value as a unifying tool in the study of special functions.

From a comparative standpoint, the Mellin transform shows clear advantages in problems where multiplicative convolution plays a central role. While the Fourier transform is better suited for additive convolution and frequency-domain analysis, and the Laplace transform for time-dependent processes, the Mellin transform excels in situations involving scaling and self-similarity [4]. This distinction explains its successful application in fields such as fractal geometry, algorithm analysis, and scale-invariant signal processing.

In applied contexts, particularly in signal and image analysis, the Mellin transform's scaling property provides robustness against changes in signal size. The discussion of results indicates that Mellin-based representations can normalize scale variations more effectively than traditional methods, thereby improving pattern recognition and feature extraction [5]. This confirms earlier findings in engineering and computational mathematics, where the Mellin transform has been used to address scale distortion problems.

Despite its advantages, the discussion also reveals certain limitations of the Mellin transform. Its applicability is restricted to functions defined on the positive real axis and requires careful consideration of convergence conditions. Moreover, the practical implementation of Mellin-based methods often involves complex analysis techniques that may pose challenges for numerical computation [6]. These limitations suggest that further research is needed to develop efficient numerical algorithms and hybrid methods that combine the Mellin transform with other analytical tools.

Overall, the discussion underscores the theoretical and practical significance of the Mellin integral transform in modern mathematics. By bridging pure mathematical theory and applied problem-solving, the Mellin transform continues to provide valuable insights across multiple disciplines. The findings of this study support existing literature and highlight the potential for further applications and methodological advancements in areas where scale invariance and asymptotic behavior play a crucial role [7; 8].

Conclusion

This study has examined the Mellin integral transform as a powerful analytical tool and highlighted its theoretical foundations as well as its wide range of applications. The results demonstrate that the Mellin transform provides a natural and effective framework for analyzing functions defined on the positive real axis, particularly in problems involving scale invariance, multiplicative structures, and asymptotic behavior. Its unique properties distinguish it from other classical integral transforms and underline its importance in modern mathematical analysis.

The findings confirm that the Mellin transform plays a significant role in simplifying the evaluation of improper integrals and in the analysis of special functions. By transferring complex analytical problems into a form suitable for contour integration and algebraic manipulation, the transform offers clear structural insights and efficient solution techniques. This makes it especially valuable in areas such as asymptotic analysis, mathematical physics, and analytic number theory.

Furthermore, the study illustrates the relevance of the Mellin transform in applied mathematics and engineering. Its ability to handle scaling effects and self-similar structures enables effective applications in signal and image analysis, algorithmic complexity, and scale-invariant pattern recognition. These applications demonstrate that the Mellin transform is not only a theoretical construct but also a practical tool for addressing real-world analytical problems.

In conclusion, the Mellin integral transform represents a versatile and robust method with both theoretical depth and practical significance. Future research may focus on the development of efficient numerical implementations and on the integration of Mellin-based techniques with other analytical methods to expand its applicability. Such advancements will further enhance the role of the Mellin transform in contemporary mathematical and applied research.

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