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METHODOLOGICAL FOUNDATIONS FOR TEACHING MAXWELL'S EQUATIONS BASED ON THE ELECTROMAGNETISM SECTION

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It is well known that electromagnetism is one of the most complex and, at the same time, fundamental branches of physics. The phenomena studied within this field form the basis of scientific and technological progress. Electric generators, electric motors, transformers, and many other devices that operate according to the laws of electromagnetism have become an integral part of everyday human life. Therefore, a deep understanding of this subject by students is of paramount importance, both for comprehending the principles of modern technology and equipment and for broadly expanding their scientific outlook.

It is known that there are textbooks that serve as good sources for students to study this section [1–8]. These sources, depending on the future profession of the students, present the fundamentals of this field in the required depth and manner suitable for each specialty.

Currently, new pedagogical technologies and methods are the main tools of education, and their use in teaching is considered a guarantee of the quality of learning. One such approach, in our view, is the testing of students' knowledge and the analysis of the fundamentals and laws of the Electromagnetism section from the perspective of Electrodynamics, more precisely, from the standpoint of the system of Maxwell's equations. From this perspective, we believe that work [9] is an excellent source. In this work, the fundamentals of Electrodynamics are presented in a way that makes it easy to identify the connection with the Electromagnetism section. Below, we provide such an analysis of the fundamentals and laws of these sections.

As is known, Electrodynamics is the science of the electromagnetic field and its properties. The system of Maxwell's equations serves as the mathematical framework of Electrodynamics. The system of Maxwell's equations (or the system of electromagnetic field equations) for vacuum in differential form is written as follows:

$$\begin{cases} div\,\vec{E}=\frac{\rho}{\varepsilon_0} \\ rot\vec{E}=-\frac{\partial\vec{B}}{\partial t} \\ div\vec{B}=0 \\ rot\vec{B}=\mu_0\vec{j}+\varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}. \end{cases} \quad (1)$$

Here, $\boldsymbol{\varepsilon_0}$ is the electric constant, $\boldsymbol{\mu_0}$ is the magnetic constant, $\boldsymbol{\rho}$ is the volume charge density, and $\vec{j}$ is the current density. The electric and magnetic constants characterize the electrical and magnetic properties of vacuum, respectively. Therefore, this system of equations describes the electromagnetic field in vacuum, i.e., it does not include parameters characterizing matter (medium).

In this system of equations, each equation has its own physical meaning. To understand the essence of each equation, it is necessary to clarify the meaning of mathematical operators such as gradient (*grad*), divergence (*div*), and curl (*rot*). Note that these operators also appear in the Electromagnetism section—for example, in the equations of electrostatics and magnetostatics, or in the relation between the field intensity and the scalar potential of the electric field, i.e., $\vec{E}=-grad\varphi$.

It is known that the divergence of a field represents its source function, while the curl characterizes the nature of the field [10,11]. If the divergence of a field is not zero, the field has a source; if the curl is not zero, the field is vortex-like, i.e., the field lines are closed; if the curl is zero, the field is irrotational, i.e., the field lines are not closed.

Accordingly, the Maxwell's equations system (1) can be interpreted as follows:

1. The source of the electric field is electric charge.
2. A varying magnetic field generates a varying electric field, which is also vortex-like.
3. Magnetic charges do not exist in nature.
4. In addition to electric current, a varying electric field also creates a varying magnetic field, which is vortex-like.

On the other hand, the right-hand side of the second equation and the second term in the fourth equation also reflect the phenomenon of Faraday's electromagnetic induction, according to which varying electric and magnetic fields generate each other.

Thus, explaining the mathematical and physical meanings of the gradient, divergence, and curl operators helps students understand the essence of the equations in the Maxwell system. This, in turn, allows them to comprehend the fundamentals of this section as a whole and to recall and reinforce the basics of Electromagnetism.

When presenting the Maxwell's equations system to students, we believe attention should be given to the following points: First, the source of the electromagnetic field is a system of charged particles. Electric charge is the fundamental concept of Electrodynamics, i.e., charged particles or a system of charged particles create an electromagnetic field around themselves. The nature of the electric charge itself is still not fully understood. Electric charge is considered an intrinsic property of elementary particles—it exists together with the particle and does not exist independently of it. From the perspective of modern particle physics, the properties of elementary particles are described in 11-dimensional space. The electric charge of hadrons is determined according to the Gell-Mann–Nishijima relation [12,13]:

$$Q = T_3 + \frac{Y}{2} = T_3 + \frac{B+S}{2}, \quad (2)$$

Here, $T_3 -$ is the third component of isospin, and $\boldsymbol{Y}$ is the hypercharge, which is defined as the sum of $\boldsymbol{B}$ – the baryon number, and $\boldsymbol{S}$ – the strangeness quantum number. Currently, the hypercharge has been extended to include the newly discovered $\boldsymbol{c}$, $\boldsymbol{t}$, and $\boldsymbol{b}$ quarks:

$$Y = B + S + C + B' + T$$

Here, $\boldsymbol{C}$ is the charm quantum number, $\boldsymbol{B'}$ is the beauty (or bottom) quantum number, and $\boldsymbol{T}$ is the topness quantum number. These quantum numbers- $\boldsymbol{T_3}$, $\boldsymbol{B}$, $\boldsymbol{S}$, $\boldsymbol{C}$, $\boldsymbol{B'}$, T—are also called charge quantum numbers, reflecting the complex nature of electric charge.

Second, magnetic charges, whose existence was predicted by the eminent English physicist P. Dirac in the 1930s and are named Dirac monopoles in his honor, have not yet been experimentally confirmed [14–16]. It is assumed that Dirac magnetic monopoles can have two signs, positive and negative, similar to electric charges. If these monopoles are experimentally discovered, the third Maxwell equation would require modification.

In the Maxwell equations system, if we apply the condition of stationarity, i.e., $\rho = const$, $\vec{j} = const$, and $\frac{\partial \vec{E}}{\partial t} = 0$, $\frac{\partial \vec{B}}{\partial t} = 0$, we obtain the system of electrostatics (3) and magnetostatics (4) equations of the Electromagnetism section.

$$\begin{cases} div\vec{E} = \frac{\rho}{\varepsilon_0} \\ rot\vec{E} = 0 \end{cases} \quad (3)$$

$$\begin{cases} div\vec{B} = 0 \\ rot\vec{B} = \mu_0 \vec{j} \end{cases} \quad (4)$$

It is known that the system of equations (3) describes the electrostatic field. This field is irrotational (the field lines are not closed), i.e., the lines of the field begin on positive electric charges and end on negative ones.

$$rot\vec{E}=0.$$

The system of equations (4) describes the magnetostatic field. This field remains vortex-like, i.e., the lines of the vector field are closed and have neither a beginning nor an end; therefore…

$$rot\vec{B}\neq 0.$$

It is known that the Maxwell equations system is a set of first-order differential equations, the solutions of which are not easy from a practical point of view and often lead to cumbersome calculations. Therefore, there arises a need to transform the Maxwell equations system into a different form to simplify its solution.

This explains the necessity of introducing the electromagnetic field potentials $\vec{A}$ and φ. To introduce the field potentials, one can use equation (1.3) of the Maxwell equations system. From $div\vec{B}=0$ (1.3), we can define $\vec{B}=rot\vec{A}$ (5). This is because, mathematically, for any arbitrary vector $\vec{a}$, $divrot\vec{a}=0$, i.e., the divergence of the curl of any arbitrary vector $\vec{a}$ is zero, and therefore relation (1.3) is satisfied. Here, $\vec{A}=\vec{A}(\vec{r};t)$ is called the vector potential of the electromagnetic field.

If we substitute equation (5) into equation (1.2) of the Maxwell equations system, we obtain the following expression:

$$rot\vec{E}=-\frac{\partial}{\partial t}rot\vec{A}=-rot\frac{\partial\vec{A}}{\partial t}.$$

Or

$$rot\left(\vec{E}+\frac{\partial\vec{A}}{\partial t}\right)=0.$$

The expression in the parentheses can be denoted as follows:

$$\vec{E}+\frac{\partial\vec{A}}{\partial t}=-grad\varphi. \quad (6)$$

This is because the curl of the gradient of any arbitrary scalar function is zero. Here, $\varphi=\varphi(\vec{r},t)$ is called the scalar potential of the electromagnetic field. From expression (6), the electric component of the electromagnetic field is expressed as follows:

$$\vec{E}=-grad\varphi-\frac{\partial\vec{A}}{\partial t}. \quad (7)$$

Accordingly, equations (1.3) and (1.2) of the Maxwell equations system will take the following form:

$$\vec{B}=rot\vec{A}\,(5)$$

$$\vec{E}=-grad\varphi-\frac{\partial\vec{A}}{\partial t}\,(7)$$

The remaining equations of the Maxwell equations system (equations (1.1) and (1.4)), using equations (5) and (7), will take the following form:

$$rot\vec{B}=rotrot\vec{A}=-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}-\frac{1}{c^2}grad\frac{\partial\varphi}{\partial t}+\mu_0\vec{j} \qquad (8)$$

$$div\vec{E}=\Delta\varphi+\frac{\partial}{\partial t}div\vec{A}=-\frac{1}{\varepsilon_0}\rho \qquad (9)$$

There, $\frac{1}{c^2}=\varepsilon_0\mu_0$.

Using a well-known relation from mathematics

$$rotrot\vec{a}=graddiv\vec{a}-\Delta\vec{a}$$

we obtain the following expression:

$$rotrot\vec{A}=graddiv\vec{A}-\Delta\vec{A}=-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}-\frac{1}{c^2}grad\frac{\partial\varphi}{\partial t}+\mu_0\vec{j}.$$

Or

$$\Delta\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}=-\mu_0\vec{j}+grad\left(div\vec{A}+\frac{1}{c^2}grad\frac{\partial\varphi}{\partial t}\right).$$

The last expression can be simplified if the expression in the parentheses is set to zero.

$$div\vec{A}+\frac{1}{c^2}grad\frac{\partial\varphi}{\partial t}=0 \quad (10)$$

Expression (10) is called the Lorentz gauge. From this expression, we obtain:

$$\frac{\partial}{\partial t}div\vec{A}=-\frac{1}{c^2}grad\frac{\partial^2\varphi}{\partial t^2}.$$

Using this expression, equations (8) and (9) will take the following form:

$$\begin{cases}\Delta\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}=-\mu_0\vec{j}\\ \Delta\varphi-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}=-\frac{1}{\varepsilon_0}\rho\end{cases} \quad (11)$$

By solving the system of equations (11), one can determine the electromagnetic field potentials $\vec{A}$ and φ. Using equations (5) and (7), one can then determine the components of the electromagnetic field — $\vec{E}$ and $\vec{B}$. The system of equations (11) is called the Maxwell equations in terms of potentials, or the electromagnetic field equations in terms of potentials for vacuum.

This system of equations is a set of second-order differential equations, which are well-studied in mathematics and can be solved easily. This is precisely the rationale for introducing electromagnetic field potentials and transforming the Maxwell equations into a system of second-order differential equations. $\vec{A}$ and $\boldsymbol{\varphi}$ are auxiliary parameters, while the components of the electromagnetic field — $\vec{E}$ and $\vec{B}$ — are the physical parameters.

Here, it is also important to explain the meaning of the Lorentz gauge. The electromagnetic field should have four components, but only two components are observed — the electric and magnetic components, $\vec{E}$ and $\vec{B}$. The other two components are not observed. According to the Lorentz gauge, out of the four components, two are excluded, leaving only the observable components — $\vec{E}$ and $\vec{B}$. The relationship between the electromagnetic field potentials $\vec{A}$, $\boldsymbol{\varphi}$ and the field components $\vec{E}$, $\vec{B}$ is given by relations (5) and (7).

Under the condition of stationarity, the system of equations (11) takes the following form:

$$\begin{cases}\Delta\vec{A}=-\mu_0\vec{j}\\ \Delta\varphi=-\frac{1}{\varepsilon_0}\rho\end{cases} \quad (12)$$

And equation (7) takes the following form:

$$\vec{E}=-grad\varphi \, (13)$$

It can be seen that equation (13) represents the well-known relation between the electric field intensity $\vec{E}$ and $\boldsymbol{\varphi}$ the scalar potential in the Electromagnetism section.

If, in the Maxwell equations system, we use the conditions $\rho=0$ and $\vec{j}=0$, i.e., an electromagnetic field in the absence of sources, we obtain the system of equations for electromagnetic waves.

$$\begin{cases} div\vec{E}=0 \\ rot\vec{E}=-\frac{\partial\vec{B}}{\partial t} \\ div\vec{B}=0 \\ rot\vec{B}=\varepsilon_0\mu_0\frac{\partial\vec{E}}{\partial t} \end{cases} \quad (14)$$

This means that a system of charged particles generates an electromagnetic field, and this field, after leaving its source, can exist in the form of electromagnetic waves and propagate at the speed of light. According to (14.2) and (14.4), the electric and magnetic components of the electromagnetic field generate each other and, in the form of electromagnetic waves, propagate at the speed of light. Here, it is also possible to explain the difference between the concepts of "field" and "wave," which is important for understanding the essence of these concepts, the nature of waves and fields by students, and for broadening their overall scientific outlook.
If the following substitutions are made in the Maxwell equations system for vacuum:

$$\varepsilon_0\to\varepsilon_0\varepsilon,\ \ \mu_0\to\mu_0\mu$$

then we obtain the Maxwell equations system for matter (medium)

$$\begin{cases} div\vec{D}=\rho \\ rot\vec{E}=-\frac{\partial\vec{B}}{\partial t} \\ div\vec{B}=0 \\ rot\vec{H}=\vec{j}+\frac{\partial\vec{D}}{\partial t} \end{cases} \quad (15)$$

and the following well-known relations between the quantities $\vec{E}$ and $\vec{D}$, $\vec{B}$ and $\vec{H}$

$$\vec{D}=\varepsilon_0\varepsilon\vec{E},$$
$$\vec{H}=\frac{1}{\mu_0\mu}\vec{B}.$$

Here, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$ are, respectively, the dielectric permittivity and magnetic permeability of the material (medium). From these relations, it can be concluded that $\vec{E}$ and $\vec{B}$ characterize, respectively, the electric and magnetic fields in vacuum, while $\vec{D}$ and $\vec{H}$ characterize, respectively, the electric and magnetic fields in a material (medium).

From the Maxwell equations system, one can also obtain the relation between $\boldsymbol{c}$ — the speed of light — and the electrodynamic parameters $\boldsymbol{\varepsilon_0}$ and $\boldsymbol{\mu_0}$ (i.e., the connection with the Optics section).

$$\frac{1}{c^2}=\varepsilon_0\,\mu_0\ .$$

As well as the expression for the speed of light in a material (medium), which has the form: $c\ =\dfrac{c}{\sqrt{\varepsilon\,\mu}}$,

where,

$\sqrt{\varepsilon\mu}=n-$ the refractive index of the material (medium).

Thus, the use of the mathematical framework of Electrodynamics — the Maxwell equations system — is a valuable tool not only for consolidating knowledge in electromagnetism but also for ensuring a thorough understanding of the Electrodynamics section by students. According to the curriculum, students study Electrodynamics after completing the

Electromagnetism section. This allows students to recall the fundamentals of the previous course and analyze electromagnetic processes from the perspective of Electrodynamics, specifically through the Maxwell equations system.

On the other hand, the topic "Electromagnetic Induction" is one of the most complex and difficult for students to master. Therefore, it is advisable to review this topic again in its various manifestations, including mutual and self-induction. Here, we have presented only some aspects of such an analysis, while in practice, the correspondence between these sections can be demonstrated much more extensively according to their curricula.

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