

COMBINATORICS PROBLEMS

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Abstract

This article presents formulas related to combinatorics and demonstrates solutions to combinatorics problems using these formulas. The problems discussed in this article help to increase students' interest in mathematics, as well as to develop logical thinking and problem-solving skills in combinatorics.

Keywords

combinatorics, formula, set, factorial.

INTRODUCTION

In this article, we will get acquainted with a general concept of combinatorics, elements of combinatorics, and several combinatorics problems. Studying combinatorics problems — in addition to reinforcing the topic and developing problem-solving skills — teaches students to think deeply, to solve real-life problems, and to reason logically.

To solve a number of practical problems, it is necessary to select elements from a given set that have a certain property and arrange them in a specific order.

Definition. Problems related to selecting subsets consisting of elements with a certain property from a finite set, or to arranging the elements of a set in a specific order, are called combinatorial problems.

For example, the following are combinatorial problems: how many ways can a team of four workers be formed from ten workers (production organization); how atoms can combine in a molecule (chemistry); how amino acids can be arranged in proteins (biology); how blocks can be combined in and rious orders in a multi-block mechanism (engineering design); rotating different crops on several field plots (agronomy); distribution of the state budget among production sectors (economics) — all of these demonstrate the application of combinatorics in and rious fields of human activity.

Definition. The branch of mathematics that deals with combinatorial problems is called combinatorics.

Combinatorics was first studied as an independent science by the German mathematician G. Leibniz, who published the work "On the Art of Combinatorics" in 1666.

In combinatorics, there are two fundamental rules called the addition rule and the multiplication rule.



Addition rule. If a certain α choice can be made in $m(\alpha)$ ways, and β choice can be made in $m(\beta)$ ways, and α choice differs from the β choice differs from the α or β choice can be made in is $m(\alpha \text{ or } \beta) = m(\alpha) + m(\beta)$ formula.

Multiplication rule. If a certain α choice can be made in $m(\alpha)$ ways, and β choice can be made in $m(\beta)$ ways, then the number of ways to make the “ α and β ” choice (i.e., the pair (α, β)) is $m(\alpha \text{ and } \beta) = m(\alpha) \cdot m(\beta)$ formula.

One of the most commonly used concepts in solving combinatorial problems is the concept of permutation.

Definition. For a finite set of n elements, forming a subset by changing only the arrangement order of all elements is called a n -element permutation.

The number of permutations of a given n elements is denoted P_n .

Theorem. The number of permutations of n elements is computed by the formula $P_n = n!$.

Here $n!$ is read as "n factorial" and $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n$ is defined as above. Here $0! = 1$ by convention. For example, $3! = 1 \cdot 2 \cdot 3 = 6$, $4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$, $5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$ and so on. When computing factorials, it is convenient to use $(n+1)! = n! \cdot (n+1)$. For example, for $n=3$ -element set $\{a, b, c\}$ the permutations formed $\{a, b, c\}$, $\{b, a, c\}$, $\{c, b, a\}$, $\{a, c, b\}$, $\{b, c, a\}$, $\{c, a, b\}$ and their number is $P_3 = 3! = 1 \cdot 2 \cdot 3 = 6$.

Another combinatorial concept is the concept of combination.

Definition. For a finite set of n element set, forming k ($k < n$) -element subsets differing in at least one element is called a combination. n -element k -combination taken from.

For example, $\{a, b, c\}$ of the set $n=3$, the two-element combinations are $\{a, b\}$, $\{a, c\}$, $\{b, c\}$ and their count is 3. Here $\{b, a\} = \{a, b\}$, $\{a, c\} = \{c, a\}$, $\{b, c\} = \{c, b\}$ are considered equal. The number of

n -element k -combinations is denoted C_n^k and its value is computed by the formula $C_n^k = \frac{n!}{k!(n-k)!}$.

Using the numbers C_n^k introduced by this formula, we can write the following identity:

$$(a+b)^n = a^n + C_n^1 a^{n-1} b + C_n^2 a^{n-2} b^2 + \dots + C_n^{n-1} a b^{n-1} + b^n = \sum_{k=0}^n C_n^k a^{n-k} b^k.$$

In this identity, n is an arbitrary natural number; it generalizes $(a+b)^2$ and $(a+b)^3$ the short multiplication formulas, and is known as

Newton's binomial theorem. The numbers C_n^k are called binomial coefficients.

If in Newton's binomial we set $a=b=1$ or $a=1, b=-1$ then the following hold:
 $\sum_{k=0}^n C_n^k = 2^n, \quad \sum_{k=0}^n (-1)^k C_n^k = 0$

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If in the formula k is replaced by $n-k$ or $k=0$ or $k=n$ then $C_n^k = C_n^{n-k}$, $C_n^0 = C_n^n = 1$ hold. These simplify the computation of combinations.

The concept called arrangement (placement) is also used in solving combinatorial problems.

Definition. For a finite set of n elements, forming subsets that differ in either elements or their arrangement order is called an arrangement. k n -element k -arrangement.

The number of permutations of a given n -element k -arrangements is denoted A_n^k and its value is computed by

$$A_n^k = n(n-1)(n-2)\cdots[n-(k-1)] \text{ or } A_n^k = \frac{n!}{(n-k)!}.$$

For example, $\{a,b,c\}$ $n=3$ -element $k=2$ -arrangements are $\{a;b\}$, $\{b;a\}$, $\{a;c\}$, $\{c;a\}$, $\{b;c\}$, $\{c;b\}$ and their number is $A_3^2 = 3 \cdot 2 \cdot 1 = 6$ or $A_3^2 = \frac{n!}{(n-k)!} = \frac{3!}{(3-2)!} = \frac{6}{1} = 6$.

Solved Problems on the Topic

1. A company employs 10 male and 8 female workers. In how many ways can one worker be selected from this company?

Solution: α — selecting a male worker, β — selecting a female worker. Then,, by the condition,, $m(\alpha)=10$, $m(\beta)=8$ one worker can be selected in $m(\alpha \text{ or } \beta) = m(\alpha) + m(\beta) = 10 + 8 = 18$ ways.

2. A group of 10 students received two vouchers. In how many ways can these vouchers be distributed?

Solution: α represents distributing the first voucher, and β the second voucher. Then $m(\alpha)=10$ and $m(\beta)=9$, since

when one student receives the first voucher, nine remain for the second. Therefore, the number of ways equals $m(\alpha \text{ and } \beta) = m(\alpha) \cdot m(\beta) = 10 \cdot 9 = 90$.

3. A construction site employs 10 plasterers and 8 painters. In how many ways can a pair of one plasterer and one painter be selected?

Solution: $m(\alpha)=10$ and $m(\beta)=8$ we get $m(\alpha \text{ and } \beta) = m(\alpha) \cdot m(\beta) = 10 \cdot 8 = 80$.

4. A quality inspector must check 5 products manufactured at a company one after another. In how many ways can the inspector do this?

Solution: Checking 5 products one after another consists of permutations of 5 elements.

That is, $P_5 = 5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$.

5. A group of five people was assigned to inspect a production company. In how many ways can a subgroup of three people be formed from these five?



Solution: $C_n^k = \frac{n!}{k!(n-k)!}$ formula. We have $n=5$, $k=3$ we get
 $C_5^3 = \frac{5!}{3!(5-3)!} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 2 \cdot 3 \cdot 2!} = \frac{4 \cdot 5}{1 \cdot 2} = \frac{20}{2} = 10.$

6. An employee at a garment factory was given the option to choose any two days of the week as days off. In how many ways can the employee choose the days off?

Solution: Representing the days of the week as the $n=7$ -element set $\{1,2,3,4,5,6,7\}$ the days off form pairs such as $\{1,2\}$, $\{1,3\}$, $\{1,4\}$,... . Here $\{i,j\}$ and $\{j,i\}$ represent the same option. Therefore, choosing days off forms $n=7$ -element $k=2$ -combinations and their number is
 $C_7^2 = \frac{7!}{2!(7-2)!} = \frac{7!}{2!5!} = \frac{6 \cdot 7}{1 \cdot 2} = \frac{42}{2} = 21 .$

7. A student decided to allocate one day of the week for additional preparation for each of 4 subjects. In how many ways can the student distribute the days of the week among the subjects?

Solution: Let the days chosen by the student for subjects I-IV be represented as a $k=4$ -element set $X=\{x_1, x_2, x_3, x_4\}$ and the days of the week as $n=7$ -element set $H=\{1,2,3,4,5,6,7\}$. Then $X \subset H$, and selecting from $n=7$ elements as $k=4$ -arrangements, since the order of assignment matters. For example, $\{2,4,6,7\}$ assigns Monday (2) to subject I, Wednesday (4) to II, Friday (6) to III, Saturday (7) to IV. Thus $\{4,2,6,7\}$, $\{6,4,2,7\}$ represent different assignments. Therefore, the student can assign the days of the week to the subjects in

$$A_7^4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = 4 \cdot 5 \cdot 6 \cdot 7 = 840 \text{ ways.}$$

8. The first-year English program at the Faculty of Foreign Languages has 10 subjects, and 4 subjects are taught, and 4 different classes are held each day. In how many ways can the daily schedule be arranged?

Solution: All possible daily class schedules consist of arrangements of 10 elements taken 4 at a time. Using $A_n^k = \frac{n!}{(n-k)!}$ formula. We have $n=10$, $k=4$ we get

$$A_{10}^4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} = 7 \cdot 8 \cdot 9 \cdot 10 = 5040.$$

9. If each integer is expressed using three distinct digits, how many such integers can be formed?

Solution: The desired number consists of arrangements of 3 taken from 9 meaningful digits. That is,

$$A_9^3 = \frac{9!}{(9-3)!} = \frac{9!}{6!} = 7 \cdot 8 \cdot 9 = 504.$$

By using $A_n^k = n(n-1)(n-2) \cdots [n-(k-1)]$ This can also be found using the formula directly: $A_9^3 = 9 \cdot 8 \cdot 7 = 504.$

10. On the shelf there are 5 different types books and. In how many gifts can be made out of it, including 7 books?



Solution. From this $\overline{C_n^m} = C_{m+n-1}^m = \frac{(m+n-1)!}{m!(n-1)!}$ we get:

$$\overline{C_5^7} = C_{5+7-1}^7 = C_{11}^7 = \frac{11!}{7!(11-7)!} = \frac{11!}{7!4!} = 330.$$

Answer. 330.

11. On two mutually parallel lines a and b lines, four points are taken on one and three on the other. How many triangles can be formed with vertices at these points?

Solution. a two of the four points on line $C_4^2 = \frac{4!}{2!2!} = 6$ ways, and b $6 \cdot 3 = 18$ triangles can be formed. b two of the three points on line $C_3^2 = 3$ ways, and a $3 \cdot 4 = 12$ triangles can be formed. Therefore, in total $18 + 12 = 30$ triangles can be formed.

Answer. 30.

12. On two mutually parallel lines a and b lines, five points on one and four on the other. How many triangles can be formed with vertices at these points?

Solution. one vertex of the triangle is on a line; two vertices must be on line b . So a points taken one at a time and b points taken two at a time: $C_5^1 \cdot C_4^2 = 5 \cdot 6 = 30$. Now if one vertex is on b line and two vertices are on line a : $C_5^2 \cdot C_4^1 = 10 \cdot 4 = 40$.

Therefore, in total $30 + 40 = 70$ triangles can be formed.

Answer. 70.

13. How many seven-digit numbers have the digit 2 appearing 3 times and digit 3 appearing 4 times and digit

Solution. In the seven-digit number, three digits equal 2 and the remaining four equal 3, so we compute: $\frac{7!}{3!4!} = 35$.

Answer. 35.

14. In how many ways can a twenty-volume set be shelved so that volumes one and two are not adjacent?

Solution. A twenty-volume set can be arranged on the shelf in $20!$ ways in total. With the condition that volumes one and two are not adjacent, the number of arrangements is $20! - 2 \cdot 19! = 18 \cdot 19!$ ways.

Answer. $18 \cdot 19!$.



15. By rearranging the letters in the word "Gazeta," how many different words (letter sequences) can be formed?

Solution. By rearranging six different letters, $6!$ words can be formed; since "a" appears twice in "Gazeta," $6!$ using $2!$ giving $\frac{6!}{2!} = 360$ different words.

Answer. 360.

Exercises for Independent Solving:

1. Find the values of the following expressionsngtopilsin:

1) $\frac{14!}{12!}$; 2) $\frac{16!}{18!}$; 3) $\frac{9!}{5! \cdot 4!}$; 4) $8! + 9!$.

2. Prove the following:

1) $\frac{(m+3)!}{m!} = (m+1)(m+2)(m+3)$; $\frac{n!}{(n-m)!} = n(n-1) \cdots (n-m+2)(n-m+1)$, where $n > m$.

3. Perform the operations:

1) $\frac{1}{n!} - \frac{1}{(n+1)!}$; 2) $\frac{1}{(k-1)!} - \frac{1}{k!}$.

4. How many nine-digit numbers can be written using nine distinct digits?

Answer: 362880.

5. In how many ways can 12 people be seated at a table set for 12?

Answer: 479001600.

6. 6 students are competing. In how many ways can the places be distributed among them?

7. In how many ways can a student select 4 books from 6?

8. For work in a certain department, 6 workers must be selected from 20. In how many ways can this be done?

9. Prove the and lidity of the identity:

1) $C_7^4 + C_7^3 = C_8^4$; 2) $C_{10}^5 + C_{10}^6 = C_{11}^6$.

10. Simplify the expression:

$$\frac{3}{2(2n-1)} C_n^{2n-3}.$$

11. 12 teams participate in a competition. In how many ways can three different medals be distributed?

Answer: $A_{12}^3 = 1320$.

12. There are 30 students in a group. 3 of them need to be selected to work on a computer. In how many ways can this be done?

Answer: $C_{30}^3 = 4060$.

13. There are 5 rolls of fabric of different colors. How many different five-colored flags can be made so that each fabric occupies exactly one stripe?

Answer: $P_5 = 5! = 120$.



14. Solve the equation:

1) $\frac{P_{n+2}}{P_n} = 72$; 2) $A_x^4 = A_{x-2}^2$.

Answer: 1) 7; 2) \emptyset .

CONCLUSION.

It is known that studying combinatorics problems presents some difficulty for students. If students are taught more examples like those above and practice them repeatedly, they can achieve the same level of knowledge, skills and can achieve the same level of knowledge and skills as in other topics. The problems discussed in this article also lay the groundwork for students to study problems in probability theory.

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