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CONSTRUCTION OF BASIS FUNCTIONS FOR FINITE ELEMENT METHODS

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Abstract. In this work, we study the problem of constructing basis functions for finite element methods. In this case, we use the coefficients of the algebraic-trigonometric optimal interpolation formula constructed using the Sobolev method in the Hilbert space of differentiable functions. In addition, we will prove the theorem expressing the main property of these local basis functions.

Keywords. Basis functions, ordinary differential equation, boundary value problem, finite element, optimal interpolation formula, error functional, Hilbert space.

In this work, basis functions for finite element methods are constructed. Applying these basic functions, it is possible to solve the boundary value problems set for ordinary differential equations.

At the work [1] in the Hilbert space

$$K_{2,\omega}^{(3)}=\left\{\varphi:[0,1]\to R\,|\,\varphi''-\text{abs. con. and }\varphi''' \quad L_2(0,1)\right\},$$

the optimal interpolation formula of the form

$$\varphi(x)\cong P_\varphi(x)=\sum_{\beta=0}^{N} C_\beta(x)\ \ \varphi(x_\beta)$$

is constructed. We construct a set of basis functions using the coefficients of this optimal interpolation formula.

From the coefficients of the optimal interpolation formula (1) at $N=2$ with nodes $x_0=0, x_1=\frac{1}{2}, x_2=1$, we have the following functions

$$C_0(z)=\frac{\sin(\frac{\omega z-\omega x_2}{2})\sin(\omega z-\omega x_1)}{\sin(\frac{\omega x_0-\omega x_2}{2})\sin(\omega x_0-\omega x_1)},$$

$$C_1(z)=\frac{\sin(\omega x_0-\omega z)\sin(\omega z-\omega x_2)}{\sin(\omega x_0-\omega x_1)\sin(\omega x_1-\omega x_2)},$$

$$C_2(z)=\frac{\sin(\frac{\omega x_0-\omega z}{2})\sin(\omega z-\omega x_1)}{\sin(\frac{\omega x_0-\omega x_2}{2})\sin(\omega x_2-\omega x_1)}.$$

To geometrically represent the functions $C_0(z), C_1(z)$ and $C_2(z)$, we present their graphs for the case $\omega=1$ in Fig.1.

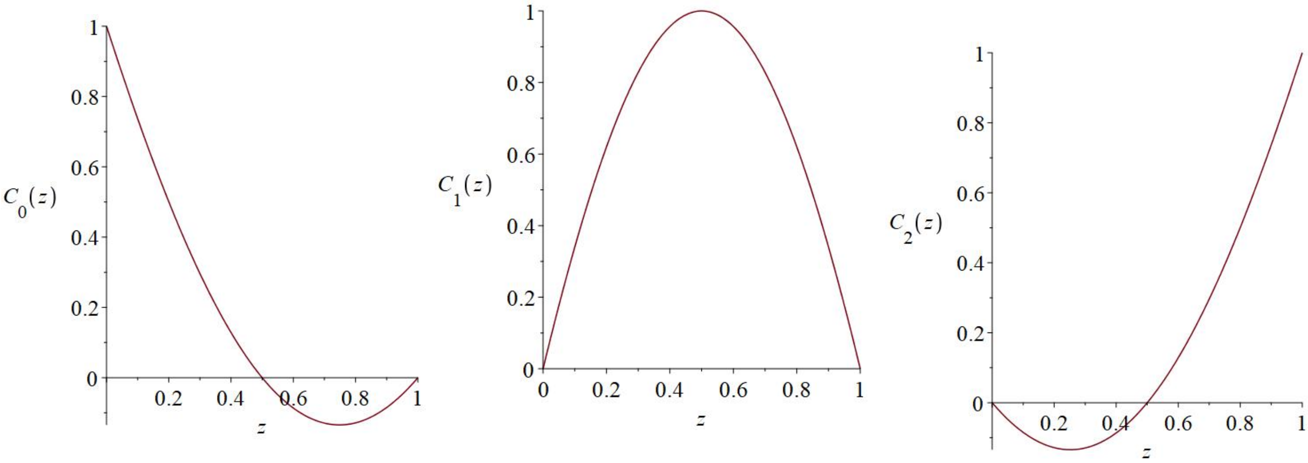

Figure 1. This figure shows the graph of the functions $C_0(z), C_1(z)$ and $C_2(z)\,(\omega=1)$.

Now we construct a set of basis functions using (2). For this, we consider the interval [0, 1] divided by $0 \quad z_0<z_1<\ldots<z_n \quad 1, z_i=ih, h=\frac{1}{n}(i=0,1,\ldots,n)$. We take basis function as follows.

Three non-zero local basis functions on the element $(y_{2i}, y_{2i+2})(i=0,1,\ldots,n-1)$ have the following form:

$$\xi_{2i}(z)=\begin{matrix} 0, & z<y_{2i-2}, \\ C_2(z), & y_{2i-2} \quad z<y_{2i}, \\ C_0(z), & y_{2i} \quad z<y_{2i+2}, \\ 0, & y_{2i+2} \quad z, \end{matrix}$$

$$\xi_{2i+1}(z)=\begin{matrix} 0, z<y_{2i}, \\ C_1(z), y_{2i} \quad z<y_{2i+2}, \\ 0, y_{2i+2} \quad z, \end{matrix}$$

$$\xi_{2i+2}(z) = \begin{cases} 0, z < y_{2i}, \\ C_2(z), y_{2i} \quad z < y_{2i+2}, \\ C_0(z), y_{2i+2} \quad z < y_{2i+4}, \\ 0, y_{2i+4} \quad z. \end{cases}$$

where

$y_{2i} = z_i$ (nodal points),

$$y_{2i+1} = \frac{z_i + z_{i+1}}{2} \text{(auxiliary points).}$$

Thus, we got a set of local trigonometric basis functions

$\xi_i(z)(i = 0,1,\ldots,2n)$.

Theorem 1. The functions $\xi_i(z)(i = 0,1,\ldots,2n)$ satisfy the following conditions:

$i)$ $\xi_i(z)(i = 0,1,\ldots,2n)$ functions are piecewise-linear functions in $[0,1]$;

$ii)$ $\xi_i(z) \quad C[0,1](i = 0,1,\ldots,2n)$;

$iii)$ $\xi_i(z)(i = 0,1,\ldots,2n)$ functions are satisfying the relation

$$\xi_i(z_j) = \begin{cases} 1, i = j, \\ 0, \text{otherwise}, \end{cases}$$

i.e.,

$$\xi_i(z_j) = \delta_{ij}\left(i = 0,1,\ldots,2n,\ j = 0,1,\ldots,2n\right),$$

where δ_{ij} is Kronecker symbol.

The proof of this theorem follows from the definition of the basis functions $\xi_i(z)(i = 0,1,\ldots,2n)$ [2].

In our next works, we will study the properties, approximation procedure and practical applications of these basis functions.

Conclusion

In this paper, local trigonometric basis functions for finite element methods are constructed. For this, the coefficients of the algebraic-trigonometric optimal interpolation formula constructed in the Hilbert space were used. Also, the theorem expressing the main property of these basis functions was proven.

References.

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