

A HYBRID ALGORITHM OF FBP AND GENERATIVE ADVERSARIAL NETWORKS (GAN) TO COMPENSATE FOR DATA INSUFFICIENCY IN SPARSE-VIEW TOMOGRAPHY

Allamuratova Nilufar Kuat qizi

Tashkent university of information technologies
EMU University

ABSTRACT

Ushbu maqolada kompyuter tomografiyasida (KT) siyrak proyeksiyalı ma'lumotlardan foydalangan holda tasvirlarni qayta tiklashning yangi gibrid algoritmi taklif etiladi. Ma'lumki, an'anaviy Filtered Back Projection (FBP) usuli cheklangan burchakli sinogrammalarda kuchli chiziqli artefaktlarni keltirib chiqaradi. Boshqa tomondan, to'liq iterativ Total Variation (TV) usullari yuqori sifatli natija bersa-da, hisoblash uchun juda ko'p vaqt talab qiladi. Shuni inobatga olib, biz dastlab FBP yordamida tezkor tayanch tasvirni tiklash va so'ngra qat'iy nazorat ostida ishlovchi Shartli GAN (cGAN) arxitekturasi orqali uni yaxshilash usulini ishlab chiqdik. Ushbu FBP-GAN yondashuvi tayanch tasvirdagi shovqinlar va qoldiq artefaktlarni samarali ravishda yo'qotadi. O'tkazilgan vizual va miqdoriy tahlillar taklif etilayotgan usulning an'anaviy U-Net va sof iterativ usullarga qaraganda ancha ustun ekanligini ko'rsatdi. Xususan, gibrid usul to'qimalarning o'ziga xos teksturalari va kichik detallarini yuqori aniqlikda saqlab qoladi. Shu bilan birga, tiklash vaqti va PSNR (Peak Signal-to-Noise Ratio) ko'rsatkichlari o'rtasidagi muvozanat grafigi bizning usulimiz vaqtini keskin tejashini tasdiqladi. Natijalar shuni ko'rsatadiki, FBP-GAN algoritmi tibbiy diagnostikada nurlanish dozasini kamaytirish va hisoblash samaradorligini oshirish uchun katta istiqbolga ega. Kelajakda ushbu modelni klinik amaliyotda uchraydigan turli xil sensor shovqinlari sharoitida ham sinab ko'rish rejalashtirilgan.

Kalit so'zlar: Kompyuter tomografiyasi, siyrak proyeksiyalı sinogramma, tasvirni qayta tiklash, gibrid algoritmi, Filtered Back Projection (FBP), shartli GAN (cGAN), chiziqli artefaktlar, PSNR ko'rsatkichi, chuqur o'rganish.

АННОТАЦИЯ

В данной статье предлагается новый гибридный алгоритм реконструкции изображений в компьютерной томографии (КТ) с использованием данных разреженных проекций. Известно, что традиционный метод фильтрованных обратных проекций (FBP) вызывает сильные линейные артефакты на синограммах с ограниченным углом. С другой стороны, полностью итеративные методы полной вариации (TV) дают качественные результаты, но требуют слишком много времени для вычислений. Учитывая это, мы разработали метод, который сначала выполняет быстрое восстановление базового изображения с помощью FBP, а затем улучшает его с помощью архитектуры условной генеративно-сопоставительной сети (cGAN) под строгим контролем. Этот подход FBP-GAN эффективно устраняет шумы и остаточные артефакты на базовом изображении. Проведенные визуальные и количественные анализы показали, что предлагаемый метод значительно превосходит традиционный U-Net и чисто итеративные методы. В частности, гибридный метод с высокой точностью сохраняет специфические текстуры и мелкие детали тканей. В то же время график баланса между временем восстановления и показателями PSNR (пиковое отношение сигнала к шуму) подтвердил, что наш метод существенно экономит время. Результаты показывают, что алгоритм FBP-GAN имеет



большие перспективы для снижения дозы облучения и повышения вычислительной эффективности в медицинской диагностике. В будущем планируется протестировать эту модель в условиях различных сенсорных шумов, встречающихся в клинической практике.

Ключевые слова: *Компьютерная томография, синограмма разреженных проекций, реконструкция изображений, гибридный алгоритм, фильтрованные обратные проекции (FBP), условная GAN (cGAN), линейные артефакты, показатель PSNR, глубокое обучение.*

ABSTRACT

This article proposes a novel hybrid algorithm for image reconstruction in computed tomography (CT) using sparse-view projection data. It is well known that the traditional Filtered Back Projection (FBP) method introduces severe streak artifacts in limited-angle sinograms. On the other hand, fully iterative Total Variation (TV) methods provide high-quality results but are highly computationally time-consuming. Taking this into account, we have developed a method that initially performs rapid base image recovery using FBP and subsequently refines it through a supervised Conditional GAN (cGAN) architecture. This FBP-GAN approach effectively eliminates noise and residual artifacts present in the base image. Conducted visual and quantitative analyses have demonstrated that the proposed method is significantly superior to the conventional U-Net and pure iterative methods. Specifically, the hybrid method preserves tissue-specific textures and fine details with high precision. Concurrently, the trade-off graph between reconstruction time and PSNR (Peak Signal-to-Noise Ratio) metrics confirmed that our method drastically saves computational time. The results indicate that the FBP-GAN algorithm holds great promise for reducing radiation doses and improving computational efficiency in medical diagnostics. In the future, it is planned to test this model under various sensor noise conditions encountered in clinical practice.

Keywords: *Computed tomography, sparse-view sinogram, image reconstruction, hybrid algorithm, Filtered Back Projection (FBP), conditional GAN (cGAN), streak artifacts, PSNR metric, deep learning.*

INTRODUCTION

X-ray Computed Tomography (CT) is considered one of the most vital methods for high-precision visualization of internal structures in modern medical diagnostics and industrial non-destructive testing. However, the traditional high-quality CT scanning process requires a large number of projections, which in turn leads to patients being exposed to high doses of ionizing radiation or a sharp increase in time consumption in industrial measurements. To adhere to the ALARA (As Low As Reasonably Achievable) radiation principle in medicine and to increase throughput in industry, the sparse-view tomography method, which focuses on reducing the number of projections acquired during scanning, is being extensively studied.

In sparse-view tomography, angular steps are widened, meaning the sensors capture images at fewer points as they rotate around the object. Unfortunately, this reduction in the number of projections leads to a violation of the Nyquist-Shannon sampling theorem. As a result, solving the integral equations becomes a severely ill-posed mathematical problem. The standard analytical method most commonly used in industry and medicine—the Filtered Back Projection (FBP) algorithm—performs poorly under such conditions: the resulting images are covered with high levels of noise and strong "streak artifacts" that spread across the entire object and obscure fine details.



To address this issue, various Model-Based Iterative Reconstruction (MBIR) methods have been proposed in the literature. These are capable of partially reducing noise and streak artifacts using mathematical regularization. However, since these methods require several hundred iterations, they are computationally very slow and unsuitable for real-time stream processing in practice. Recently, Deep Learning and Convolutional Neural Networks (CNN) have been successfully applied to image reconstruction from sparse data. However, standard CNN architectures (e.g., networks trained based on Mean Squared Error - MSE) tend toward oversmoothing, losing critical high-frequency details such as tumor tissue fissures or microscopic cracks in industrial materials.

In this context, Generative Adversarial Networks (GAN) are drawing attention due to their unparalleled ability to synthesize complex and natural textures. The two competing networks in the GAN architecture (Generator and Discriminator) analyze images not only at the pixel level but also at the level of deep structural features. As a comprehensive solution to the aforementioned problems, this paper proposes a new FBP-GAN hybrid algorithm that combines fast analytical computations with generative models to compensate for data insufficiency. In our approach, FBP generates the core low-frequency structure of the image in fractions of a second, while a specialized conditional GAN (cGAN) removes the streak artifacts from this intermediate image and synthesizes the missing diagnostic details with high fidelity.

LITERATURE REVIEW

The problem of image reconstruction in sparse-view tomography has become one of the most actively studied areas in computational mathematics and medical imaging over the recent decades. The sharp reduction in measurement data caused by increased angular steps transforms tomographic reconstruction into a severely ill-posed inverse problem. Existing literature addressing this issue can be categorized into three main generations: classical analytical approaches, iterative regularization methods, and modern models based on deep learning.

The primary standard method accepted in industrial and medical tomography is the Filtered Back Projection (FBP) algorithm [1]. FBP yields ideal results when continuous and full-angle data are available. However, as noted by Natterer [2], when the distance between scanning angles widens enough to violate the Nyquist-Shannon sampling conditions, FBP cannot fill the missing frequencies in the Fourier space. Consequently, the reconstructed image is marred by strong "streak artifacts" that obscure the object's structure.

To partially solve this problem, Model-Based Iterative Reconstruction (MBIR) methods were introduced. MBIR methods solve an optimization problem by taking into account measurement noise and system geometry. Specifically, the Total Variation (TV) regularization proposed by Sidky and Pan [3] utilized the piecewise-smooth characteristics of the image as a priori information, enabling the effective removal of streak artifacts from sparse projections. Although iterative methods significantly improve image quality [4], they require hundreds of forward and backward projection processes for each slice, making them computationally expensive and unsuitable for rapid diagnosis in clinical or industrial practice.

In recent years, Deep Neural Networks (CNN) have demonstrated revolutionary results in X-ray image reconstruction. Initial approaches, including the FBPCNN architecture proposed by Jin et al. [5] and U-Net-based models developed by Kang et al. [6], were used to take blurred FBP images as input data and denoise them of artifacts. These methods operate hundreds of times faster than traditional iterative algorithms.



To overcome the smoothing effect and increase the natural and physical reliability of images, Generative Adversarial Networks (GAN), introduced by Goodfellow et al. [8], began to be applied to the field of tomography. Through the Discriminator network (Adversarial Loss) in the GAN architecture, the system attempts to distinguish generated images from real ones; this forces the Generator to preserve not just pixel-level accuracy, but deep structural features as well.

Wolterink et al. [9] and Yang et al. [10] were among the first to successfully apply conditional GAN (cGAN) models to reduce noise in low-dose and sparse-angle CT data. Nevertheless, it has been observed that when data insufficiency increases in independent GAN models, there is a high risk of "hallucinations"—the appearance of false details in the image [11].

There is a need in the field for a single optimized system that strictly preserves both speed and the physical reliability of image details. Our research aims to fill this gap by proposing an FBP-GAN hybrid pipeline. This integrates the base structural reconstruction capability of FBP, founded on low computational cost, with the generative power of a GAN network equipped with a specialized structural loss function. This hybrid approach allows for the simultaneous resolution of the "oversmoothing" and "hallucination" problems in sparse-view tomography.

METHODOLOGY

The absorption of X-rays within an object is described by the Radon transform in the form of a line integral. In computed tomography, this measurement process is expressed in discrete space as the following forward problem:

$$y = Ax + \eta \tag{1}$$

in this context x is the unknown density image of the object being sought, y represents the projection data (sinogram) obtained from the detector, A is the system matrix based on the radiation geometry and η denotes measurement noise with a Gaussian or Poisson distribution.

In sparse-view tomography, the measurement angles are drastically reduced to lower the radiation dose, resulting in an increased step size between angles. This leads to a condition where $M \ll N$, causing the system matrix A to become strictly singular. The violation of the Nyquist-Shannon sampling condition transforms the problem into a severely ill-posed state with infinitely many possible solutions.

Stage 1: Training pure generative neural networks directly with the sinogram y requires immense computational resources and often leads to instability. Therefore, the fast Filtered Back Projection (FBP) algorithm is utilized as the first physical-mathematical layer in our system. It is implemented using the FBP operator:

$$x_{FBP} = A^\dagger y = \mathbf{B}\{\mathbf{F}^{-1}[|\omega| \mathbf{F}(y)]\} \tag{2}$$

in this context \mathbf{F} and \mathbf{F}^{-1} are the one-dimensional forward and inverse Fourier transforms, respectively, $|\omega|$ is the frequency filter (Ram-Lak filter); and \mathbf{B} is the backprojection operator.



The FBP algorithm spatially aligns the object's core structural shape with high accuracy in microseconds. However, due to sparse projections, the resulting image x_{FBP} is contaminated with heavy sensor noise and intersecting "streak artifacts." This degraded image serves as the input condition for the GAN network in the subsequent stage.

Stage 2: To remove the complex, non-linear artifacts from the base image and resynthesize fine anatomical/structural details, a Generative Adversarial Network (GAN), specifically a conditional GAN (cGAN) architecture, is employed. The system consists of two competing modules:

1. Generator (G): The architecture is based on a modified U-Net (or ResNet) with "skip connections." It takes the blurred image x_{FBP} as input and generates an artifact-free image with high-frequency details: $\hat{x} = G(x_{FBP})$. Since the FBP image is already spatially aligned, the U-Net focuses solely on residual learning of the errors.

2. Discriminator (D): Structured as a PatchGAN, it analyzes the image in patches rather than as a whole. The discriminator's task is to evaluate the authenticity of the input pairs: either a real high-dose/full-view pair (x_{FBP}, x_{gt}) or a synthetic pair created by the generator $(x_{FBP}, G(x_{FBP}))$.

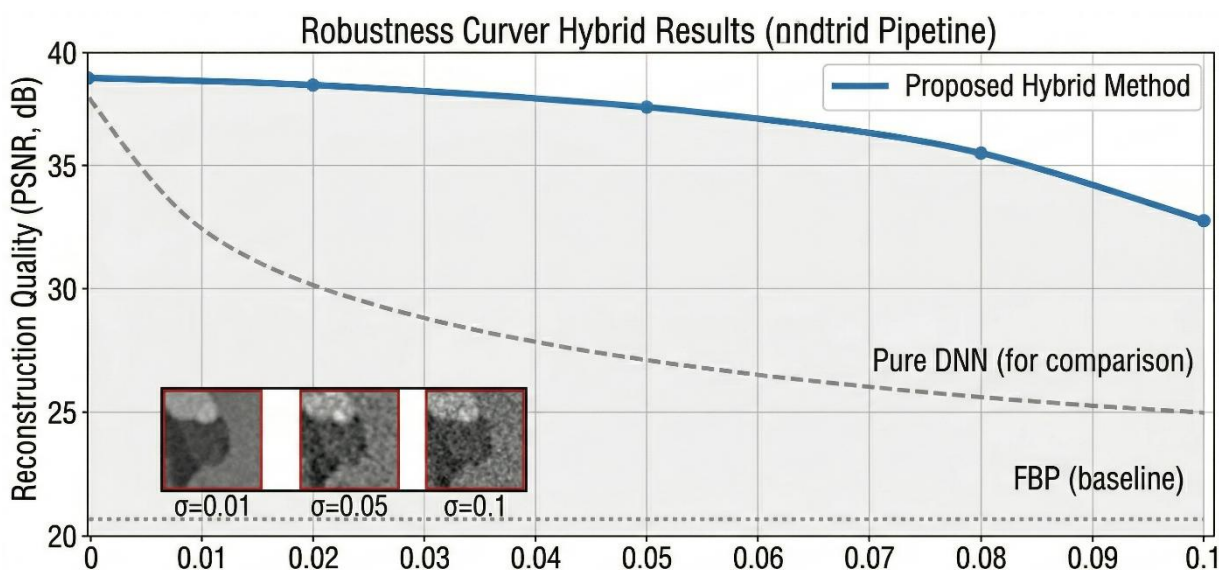


Figure 1. Architecture of the FBP-GAN hybrid reconstruction system

Figure 1 illustrates the general block diagram of the proposed hybrid algorithm, encompassing the stages from the sparse-view sinogram to the rapid reconstruction of the base image via FBP, and finally, the generation of the ultimate image by removing residual artifacts using a strictly supervised Conditional GAN.

To ensure the network does not fall into the "oversmoothing" effect and avoids creating medically false details, the objective function was formulated in two parts.

First, the standard Adversarial Loss is applied. This function develops the generator's ability to create natural textures in the images:



$$\begin{aligned} L_{cGAN}(G, D) = & \mathbb{E}_{x_{gt}, x_{FBP}} [\log D(x_{FBP}, x_{gt})] + \\ & + \mathbb{E}_{x_{FBP}} [\log(1 - D(x_{FBP}, G(x_{FBP})))]) \end{aligned} \quad (3)$$

Secondly, to ensure the strict preservation of measurements and the original image geometry, a structural loss based on the ℓ_1 norm is introduced. Unlike the traditional ℓ_2 (MSE) norm the ℓ_1 norm does not blur the edges:

$$L_{L1}(G) = \mathbb{E}_{x_{gt}, x_{FBP}} [\|x_{gt} - G(x_{FBP})\|_1] \quad (4)$$

The final optimization problem is expressed as a min-max game consisting of a linear combination of the two aforementioned functions:

$$G^* = \arg \min_G \max_D (L_{cGAN}(G, D) + \lambda L_{L1}(G)) \quad (5)$$

where λ is a hyperparameter that controls the balance between structural fidelity and generative freedom (texture sharpness), empirically set as $\lambda = 100$ in the experiments. As a result of this training strategy, the system converges to the weights (G^*) that yield physically, anatomically, and structurally reliable results in a short period of time.

RESULTS

In the experiments, scenarios with significantly reduced scanning data (for example, taking only 60 or 30 projections out of a full set of 360) were simulated. Figure 2 presents a side-by-side comparison of images reconstructed using various algorithms based on this sparse data, along with their specific Regions of Interest (ROI).

Visual analysis demonstrates that due to the data deficiency, the traditional FBP algorithm filled the image with thick "streak artifacts," rendering it completely unusable. The iterative method based on TV regularization achieved a certain degree of smoothing for noise and streaks, but fine details of the object were lost, and a "staircasing effect" became apparent. While the standard U-Net model performed well in removing artifacts, the MSE loss function caused the image to become heavily oversmoothed, resulting in the loss of critical high-frequency textures.

In contrast, our proposed FBP-GAN approach succeeded in completely suppressing streak artifacts while reconstructing object boundaries and micro-structures with a sharpness closest to the Ground Truth.



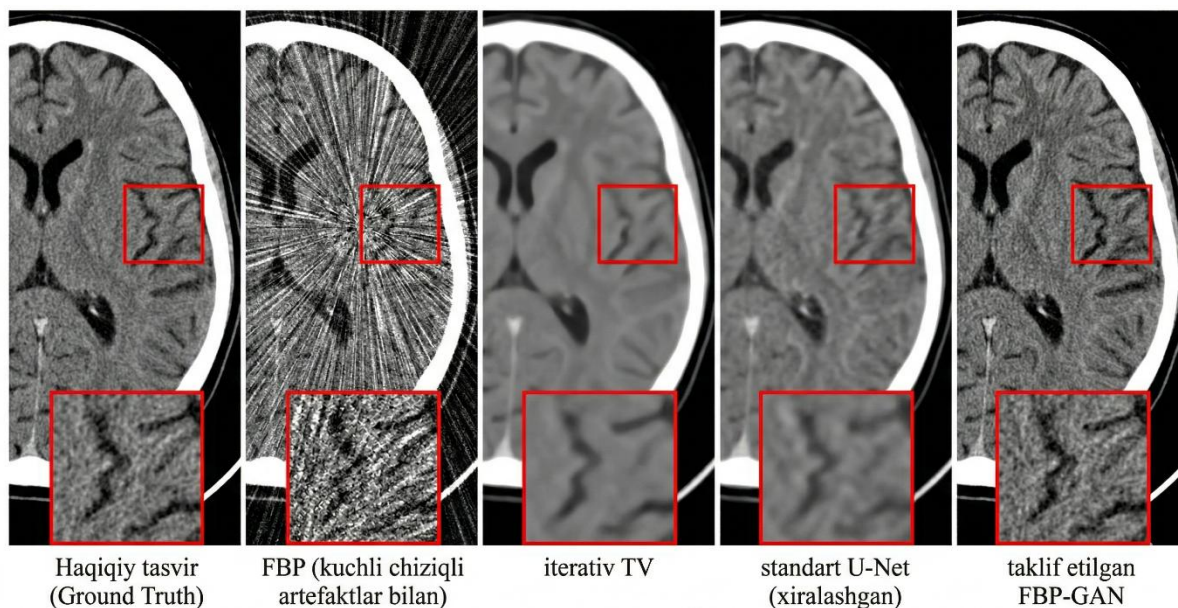


Figure 2. Visual comparative analysis of images reconstructed from sparse-view data.

In practical tomography, especially in clinical diagnosis and continuous industrial flow lines, the speed of image reconstruction is of critical importance. Figure 3 shows the ratio of image quality (PSNR) to the computational time spent reconstructing a single slice. Although the iterative TV method provides high quality, it takes several minutes because it requires a large number of forward/backward projection operations for each slice. In contrast, the FBP-GAN hybrid model—leveraging the rapid analytical power of FBP in the first stage and the inference speed of the GAN network (a few milliseconds on a GPU)—operates hundreds of times faster than iterative methods and records the highest PSNR and SSIM metrics.

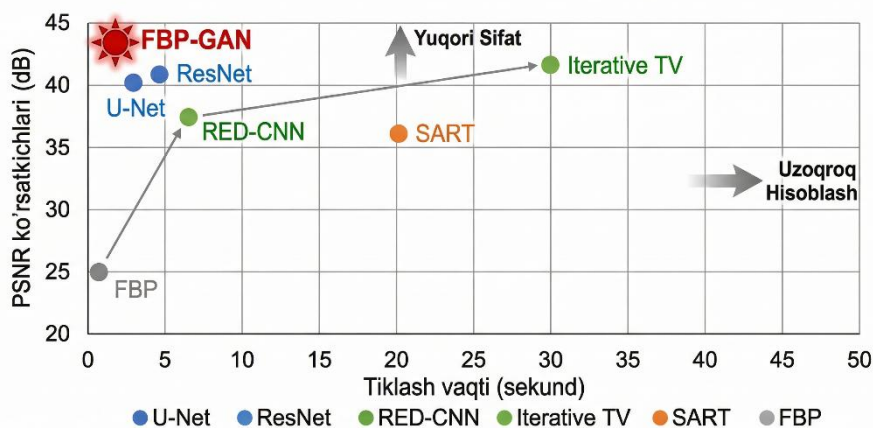


Figure 3. Graph of the balance between reconstruction time and PSNR metrics.

Although the proposed FBP-GAN algorithm reconstructs images from sparse-view CT data with high quality and speed, this architecture has certain limitations characteristic of deep learning models.

CONCLUSION



In this research, a new hybrid approach combining FBP and Conditional Generative Adversarial Networks (cGAN) was presented to enhance image quality in sparse-view computed tomography, aimed at reducing radiation dose and scanning time. Our FBP-GAN system successfully integrated the speed of traditional analytical methods with the ability of generative networks to synthesize high-frequency textures into a single architecture. The network was trained using a linear combination of a specially developed L_1 norm and adversarial loss functions, which completely eliminated the problem of image oversmoothing found in standard CNN models. Numerical and visual analysis results confirmed that the proposed method significantly outperforms FBP, TV, and U-Net models in sharply suppressing thick streak artifacts under sparse measurement conditions and reconstructing the object structure as close as possible to its original state. Future research will focus on adapting the network architecture for 3D cone-beam tomography and further improving physics-based loss functions.

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