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APPLICATION OF THE BERNOULLI DIFFERENTIAL EQUATION IN SOLVING DIFFERENTIAL EQUATIONS

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Annotation

The Bernoulli differential equation represents an important class of nonlinear first-order differential equations that can be transformed into linear equations through an appropriate substitution. This property makes it a powerful analytical tool in solving a wide range of differential equations arising in mathematics, physics, engineering, and applied sciences. The present study investigates the application of the Bernoulli equation in solving differential equations and develops a systematic approach for its use. The theoretical foundation of the Bernoulli method, its transformation mechanism, and its structural properties are analyzed in detail. Special attention is given to the conditions under which nonlinear equations can be reduced to linear form and solved using integrating factor techniques. The study also includes a detailed example illustrating the step-by-step solution procedure. The results demonstrate that the Bernoulli equation serves as an effective bridge between linear and nonlinear differential equations and provides a valuable method for solving complex mathematical problems.

Keywords: Bernoulli differential equation, nonlinear differential equation, linearization, integrating factor, substitution method, ordinary differential equations, mathematical modeling.

Introduction

The theory of ordinary differential equations is one of the central areas of mathematical analysis, focusing on equations that describe relationships between functions and their derivatives. These equations play a crucial role in modeling various natural phenomena, including mechanical systems, electrical circuits, population dynamics, and fluid motion. The ability to solve differential equations analytically is essential for understanding the qualitative and quantitative behavior of such systems.

Among the different types of first-order differential equations, nonlinear equations present significant challenges due to their complex structure. However, certain classes of nonlinear equations can be reduced to linear form using appropriate transformations. One of the most important examples of such equations is the Bernoulli differential equation.

The Bernoulli equation is defined by its characteristic nonlinear term involving a power of the dependent variable. Despite its nonlinear nature, it can be transformed into a linear differential equation by means of a substitution. This property makes it a powerful tool in both theoretical and applied mathematics.

Historically, the Bernoulli equation was introduced by Jakob Bernoulli and has since become a classical topic in the study of differential equations. It provides a clear example of how nonlinear problems can be approached using linearization techniques.

The purpose of this study is to analyze the application of the Bernoulli differential equation in solving differential equations, to present its theoretical basis, and to demonstrate its effectiveness through a detailed example.

Mathematical formulation of the Bernoulli differential equation

Consider the general form of the Bernoulli differential equation:

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

where P(x) and Q(x) are continuous functions on a given interval, and nnn is a real number such that:

$$n \neq 0, \quad n \neq 1$$

The equation is nonlinear due to the presence of the term y^n. However, it can be transformed into a linear differential equation using the substitution:

$$z = y^{1-n}$$

Transformation of the equation

Differentiating both sides with respect to x, we obtain:

$$\frac{dz}{dx} = (1-n)y^{-n}\frac{dy}{dx}$$

Multiplying the original equation by y^{-n}, we get:

$$y^{-n}\frac{dy}{dx} + P(x)y^{1-n} = Q(x)$$

Substituting $z=y^{1-n}$, the equation becomes:

$$\frac{1}{1-n}\frac{dz}{dx} + P(x)z = Q(x)$$

Multiplying both sides by 1−n, we obtain a linear differential equation:

$$\frac{dz}{dx} + (1-n)P(x)z = (1-n)Q(x)$$

Algorithm for solving the Bernoulli equation

The procedure for solving the Bernoulli differential equation consists of the following steps:

Write the equation in standard form:

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

Apply the substitution:

$$z = y^{1-n}$$

Transform the equation into a linear equation in z.

Solve the resulting linear equation using the integrating factor:

$$\mu(x) = e^{\int (1-n)P(x)\,dx}$$

Find z(x) and return to the original variable:

$$y = z^{\frac{1}{1-n}}$$

Example. Solve the differential equation:

$$\frac{dy}{dx} + y = xy^2$$

Step 1. Identify parameters

$$P(x) = 1, \quad Q(x) = x, \quad n = 2$$

Step 2. Substitution

$$z = y^{1-2} = y^{-1}$$

$$z = \frac{1}{y}$$

Differentiate:

$$\frac{dz}{dx} = -\frac{1}{y^2}\frac{dy}{dx}$$

Step 3. Transform the equation

Multiply the original equation by y^{-2}:

$$y^{-2}\frac{dy}{dx} + y^{-1} = x$$

Substitute:

$$-\frac{dz}{dx} + z = x$$

or

$$\frac{dz}{dx} - z = -x$$

Step 4. Solve linear equation

Integrating factor:

$$\mu(x) = e^{\int -1\,dx} = e^{-x}$$

Multiply both sides:

$$e^{-x}\frac{dz}{dx} - e^{-x}z = -xe^{-x}$$

$$\frac{d}{dx}(ze^{-x}) = -xe^{-x}$$

Integrate:

$$ze^{-x} = \int -xe^{-x}\,dx$$

Using integration by parts:

$$ze^{-x} = -(x+1)e^{-x} + C$$

Step 5. Final solution

$$z = -(x+1) + Ce^{x}$$

$$\frac{1}{y} = Ce^{x} - (x+1)$$

$$y = \frac{1}{Ce^{x} - (x+1)}$$

The Bernoulli differential equation plays a significant role in the theory of differential equations due to its ability to transform nonlinear problems into linear ones. This transformation simplifies the solution process and makes it possible to apply well-established methods for linear equations.

The example demonstrates that even though the original equation is nonlinear, it can be systematically reduced to a linear equation through substitution. This highlights the importance of identifying structural properties in differential equations.

The Bernoulli method is widely applicable in various fields, including physics, engineering, and mathematical modeling. It is particularly useful in solving problems involving growth and decay processes, fluid dynamics, and nonlinear systems.

However, the method requires that the solution remains nonzero in the domain of interest, and additional care must be taken when dealing with special cases or discontinuities.

Conclusion

The Bernoulli differential equation represents an important class of nonlinear first-order differential equations that can be effectively solved through linearization. The study has presented a detailed analysis of the Bernoulli method, including its theoretical foundation, transformation process, and solution algorithm.

The results show that the Bernoulli equation serves as a bridge between linear and nonlinear differential equations and provides a systematic approach for solving complex problems. The inclusion of a detailed example has demonstrated the practical applicability of the method.

Overall, the Bernoulli method remains a fundamental and powerful tool in the study of differential equations and continues to play an important role in both theoretical and applied mathematics.

References.

1. Abdullayev A., Sadullayev A., Xudoyberganov G. *Oddiy differensial tenglamalar*. Toshkent: O‘zbekiston Milliy Universiteti nashriyoti, 2018.

2. Begmatov A. *Differensial tenglamalar kursi*. Toshkent: Fan va texnologiya, 2019.

3. Malikov, Z., & Otajonova, S. (2022). ЗАДАЧА КОШИ ДЛЯ СИСТЕМ ЭЛЛИПТИЧЕСКОГО ТИПА ПЕРВОГО ПОРЯДКА В СПЕЦИАЛЬНОЙ ОГРАНИЧЕННОЙ ОБЛАСТИ В ТРЁХМЕРНОЙ ОБЛАСТИ. Science and innovation, 1(A6), 416-419.

4. Otajonova , S. (2024). APPLICATION OF ELEMENTS OF TRIGONOMETRY IN SOLUTION OF TRIANGLES. *Medicine, Pedagogy and Technology: Theory and Practice*, *2*(9), 292–304. Retrieved from

5. Otajonova, S. S. (2025). INTERACTIVE METHODS IN TEACHING MATHEMATICS TO PRIMARY SCHOOL STUDENTS: FOSTERING ENGAGEMENT AND CONCEPTUAL UNDERSTANDING. PEDAGOGIK TADQIQOTLAR JURNALI, 2(2), 84-87.

6. Shukhratovna, O. S. (2025). ORGANIZING INTERESTING GAMES FOR ELEMENTARY GRADES USING ELEMENTS OF COMBINATORICS. PEDAGOGIK TADQIQOTLAR JURNALI, 3(2), 117-119.

7. Shuxratovna, O. S. (2025). USING GAMIFICATION AND INTERACTIVE GAMES IN TEACHING MATHEMATICAL CREATIVITY. *SHOKH LIBRARY*, *1*(10).

8. Shuxratovna, O. S. (2025). DERIVATION OF FUNDAMENTAL TRIGONOMETRIC IDENTITIES. *Shokh Articles Library*, *1*(1).