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FIRST ORDER DIFFERENTIAL EQUATIONS SOLUTION METHODS AND SECOND ORDERLY DIFFERENTIAL EQUATIONS AND MECHANICAL ANALYZING MODELS

Akhmatov Azizbek Abduvakhob oglu
Navoi state University of Exact Sciences faculty mathematics department etc.
Associate Professor , PhD
aaakhmatov94@gmail.com

Isroilova Nazimakhan Salokhiddinovna
Navoi state University of Exact Sciences faculty student
nozimaisroilova234@gmail.com

Omonboyeva Muxlisa Javlonbek kizi
Navoi state University of Exact Sciences faculty student
muxlisaomonboyeva044@gmail.com

ABSTRACT: This scientific in the article first orderly and second orderly differential of equations analytical and geometric solution methods systematic accordingly is studied and their mechanic models analysis in doing importance is illuminated . First order equations for variables separation , one sexed , striped and complete differential equations methods in detail statement The second orderly linear differential equations permanent with coefficient and variable with coefficient cases for Mechanical models as spring-loaded pendulum , compulsory vibrations and dissipative systems analysis Article results differential equations theory engineering and physicist in modeling practical importance shows .

Keywords : First order differential equations , second orderly differential equations , mechanical models , variables separation , one gender equations , linear differential equations , spring pendulum , compulsory vibrations , dissipative systems , Cauchy problem , characteristic equation .

Intraduction

Differential equations science and of the technique various in fields such as mechanics , physics , biology , economics , and real processes in engineering modeling main tool is considered to be a first- order differential equations many simple dynamic systems , for example , radioactive fragmentation , populations change and cooling processes mathematician is an expression of . The second orderly differential equations and vibration movements , space of bodies orbit and electricity in chains transient processes describes [Boyce and DiPrima, 2017, p. 12].

This of the article main purpose – first and second orderly differential equations fundamental methods of solving systematization and them mechanic to models implementation to grow through theoretical knowledge practice with is to bind . In the article first in line first orderly equations for analytical methods , then second orderly linear equations and their private cases seeing Mechanical models analysis spring-loaded pendulum , pendulum and mandatory vibrations in the example of done is increased .

Literature

Differential **analysis** equations history of Isaac Newton and Gottfried Leibniz in the 17th century differential account discovery with Newton's " Philosophiæ Naturalis Principia Mathematica" contains the equations of motion differential equations in appearance [Newton , 1687, p. 89]. In the 18th century, Leonard Euler was the first orderly equations solution numerical methods working came out and second orderly linear equations for characteristic equation method offer did [Euler, 1768, p. 45].

the 19th century Augustin Louis Cauchy was the first orderly differential equations for existence and uniqueness theorems proved . These theorems differential equations theory strict mathematician basis [Coddington and Levinson, 1955, p. 74]. During this period, Joseph Louis Lagrange mechanic systems for general equations wrote and second orderly of equations private solutions in finding parameters variation method entered .

the 20th century mechanic models differential equations through analysis to do further developed . Andrei Andreevich Markov and Alexander Mikhailovich Lyapunov stagnation to the theory basis [Lyapunov , 1992, p. 132]. Modern in literature first orderly equations for variables separation , one gender equations , Bernoulli equations and complete differential equations methods wide [Zill , 2018, p. 203]. The second orderly equations for and characteristic equation , fundamental solutions system and Cauchy function such as concepts in detail analysis [Arnold , 2006, p. 156].

Mechanical models in the field spring-loaded pendulum and his/her vibrations differential equations theory classic sample is considered . Fading and mandatory vibrations are real mechanical systems important features reflection [Kreiszig , 2015, p . 278]. This article above sources synthesis did without , new examples and analytical comments with enriched .

Discussion

1. First order differential of equations solution Methods

First order simple differential equation general $F(x,y,y')=0$or $y'=f(x,y)$is given in the form of. The main analytical methods are listed below.

1.1. Separation of variables method

If the equation $\frac{dy}{dx}=g(x)h(y)$is expressed as , then the variables are separated: $\frac{dy}{h(y)}=g(x)dx$. Then both sides are integrated. For example, $\frac{dy}{dx}=xy$ the solution of the equation is $y=Ce^{x^2/2}$ will be [Boyce and DiPrima, 2017, p. 38].

1.2. Same- sex Equations

If $f(\lambda x,\lambda y)=\lambda^n f(x,y)$ the condition is met, $y=vx$ substitution leads to an equation in which the variables are separated.

1.3. Linear first orderly equations

General view : $y'+P(x)y=Q(x)$. The solution is found by the integrating multiplier $\mu(x)=e^{\int P(x)dx}$:

$$y=\frac{1}{\mu(x)}\left(\int \mu(x)Q(x)dx+C\right).$$

This method power – optional $P(x)$and $Q(x)$its functioning in functions [Zill, 2018, p. 112].

1.4. Complete differential The equation given in the form of equations

$M(x,y)dx+N(x,y)dy=0$, if$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ if , complete differential is called . In that case $u(x,y)$ The potential function is found and the solution is obtained $du=0$from $u(x,y)=C$.

This of methods every one his/her own application to the field For example , radioactive decomposition $\frac{dN}{dt}=-\lambda N$ – separation of variables, population model $\frac{dP}{dt}=aP-bP^2$ – Bernoulli equation.

2. Second orderly differential Equations

Second orderly equation general $F(x,y,y',y'')=0$or $y''=f(x,y,y')$. In this section, the main focus is on linear equations. is focused .

2.1. Permanent with coefficient linear one gender equations

Equation : $ay''+by'+cy=0$. Characteristic equation : $ar^2+br+c=0$. There are three cases:

a) Two real root $r_1 \neq r_2$: $y=C_1 e^{r_1 x}+C_2 e^{r_2 x}$.

b) Two complex root $\alpha \pm i\beta$: $y=e^{\alpha x}(C_1 \cos \beta x+C_2 \sin \beta x)$.

c) Both roots equal r: $y=(C_1+C_2 x)e^{rx}$[Kreiszig , 2015, p. 143].

2.2. Variable with coefficient linear equations

Such equations , for example , the Euler-Cauchy equation $x^2 y''+axy'+by=0$($x>0$) is in the form of, $y=x^r$ to put through solved . Other in cases special functions (Bessel, Legendre and others) are used [Arnold, 2006, p. 89].

2.3. Same- sex not been The general solution of an equation in the form of equations $ay''+by'+cy=f(x)$consists of the sum of the homogeneous solution and the particular solution. The particular solution is found by the method of undetermined coefficients or the method of variation of parameters. The method of variation of parameters is universal and $f(x)$is applicable to arbitrary [Coddington and Levinson, 1955, p. 192].

3. Mechanical models analysis to do

3.1. Spring-loaded pendulum (free) unfading vibrations) to

Hooke's law mainly , to the spring hanging mThe equation of motion for a body with mass is:

$$m\frac{d^2x}{dt^2}=-kx,$$

this on the ground k– spring stiffness. This is a second-order homogeneous linear equation. $mr^2+k=0$Complex roots from the characteristic equation. $r=\pm i\sqrt{k/m}$Solution :

$$x(t)=C_1\cos(\omega_0 t)+C_2\sin(\omega_0 t),\omega_0=\sqrt{k/m}.$$

This is harmonic. vibrations represents [Kreyszig , 2015, p. 280].

3.2 .

Dissipative vibrations If the resistance power to speed proportional if ($F_{qarsh}=-c\frac{dx}{dt}$), the equation is:

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=0.$$

Characteristic equation $mr^2+cr+k=0$. Depending on the sign of the discriminant, the types of decay (critical, subcritical or supercritical) are distinguished. For example , small in resistance (subcritical quenching) solution:

$$x(t)=e^{-\alpha t}(A\cos\beta t+B\sin\beta t),\alpha=\frac{c}{2m},\beta=\sqrt{\omega_0^2-\alpha^2}.$$

These vibrations amplitude exponential decreases [Zill, 2018, p. 267].

3.3. Mandatory Vibrations

External periodic power $F(t)=F_0\cos(\gamma t)$under the influence of:

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=F_0\cos(\gamma t).$$

Same -sex not been equation . Private solution resonance the event shows : if $\gamma$the natural frequency of the system ω_0 is close to if , vibrations amplitude sharp increases [Boyce and DiPrima, 2017, p. 345]. This mechanical in systems fatal to the consequences take arrival possible , therefore for from resonance escape in engineering important issue.

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3.4. Physical pendulum and other Models

Physics pendulum – solid of the body weight from the center impassable arrow around vibration . Movement equation :

$$I\frac{d^2\theta}{dt^2}+mgd\sin\theta=0,$$

this on the ground I– moment of inertia, d– distance from the axis to the center of gravity. Small angles for $\sin\theta\approx\theta$ The equation is linearized by replacing . At large angles, nonlinear effects occur [Arnold, 2006, p. 210].

4. Differential equations in solution numerical and geometric Analytical **methods** methods with one in line , practical in matters often numerical methods used . Euler method , Runge- Kutta methods and limited differences method first and second orderly equations for wide distributed [Kreyszig , 2015, p. 456]. Geometric point of view from the point of view , isoclines method first orderly equations solutions spatial portrait drawing opportunity gives .

Results

Below take visited of analyses main results quoted :

1. First order differential of equations four main analytical method (variables separation , one sexed , striped and complete differential) each kind in the classroom equations for effective that defined . Separation of variables method the most simple but limited ; linear equations method and integrator multiplier using wide the class cover [Zill , 2018, p. 115].
2. Second orderly linear permanent with coefficient equations characteristic equation through complete is solved . Complex roots case vibration processes , real roots and aperiodic transition processes represents .
3. Mechanical models analysis this showed that :
   a) Spring-loaded pendulum ideal harmonic vibrations gives .
   b) Fading vibrations amplitude time to pass with decrease dissipative forces in the presence event gives .
   c) Mandatory in vibrations resonance event important importance has and of the system private frequency external power frequency when approaching amplitude to infinity (idealized without) strives [Boyce and DiPrima, 2017, p. 354].
   d) Physics pendulum small vibrations in the approach linear to the model falls , big in the corners and nonlinearity important role plays .
4. Analytical methods there is not been in cases numerical methods (e.g. , Runge-Kutta) are used possible . First order equation $y'=f(x,y)$ The fourth-order Runge-Kutta method for is widely used and has high accuracy [Kreyszig, 2015, p. 460].

Conclusion

This in the article first orderly and second orderly differential of equations solution methods and their mechanic to models implementation surrounding studied . First- order equations for variables separation , one gender equations , linear equations and complete differential equations methods analysis The second orderly linear equations permanent and variable with coefficient cases for looked at . Characteristic equation method , parameters variation and undefined coefficients methods in detail illuminated . Mechanical models as spring-loaded pendulum , pendulum and mandatory vibrations and physicist pendulum in examples differential real physics of equations processes how clear description This is shown . models engineering in practice – construction structures , automobile suspensions , seismic vibrations soften systems and other in the fields wide will be applied . In the future nonlinear differential equations , partial productive differential equations and their mechanic models with related more complicated systems (e.g. , turbulent flows , elasticity theory) separately research topic to be It is also possible to use a computer modeling and digital simulations differential equations solution inseparable to the part around is going on .

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