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SOLVING THE INVERSE PROBLEM OF FINDING THE SOURCE FUNCTION IN FRACTIONAL ORDER EQUATIONS

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Abstract: In this paper , the solution of inverse problems for fractional partial differential equations in the sense of Caputo is studied. In Caputo's sense show that the solution of the partial differential equation of fractional order exists and is unique , and it is intended to obtain results regarding the correct and inverse problem of determining the source function caught.

Key words: Fractional equation, Mittag-Leffler function, Caputo fractional derivative, exact problem, inverse problem, Fourier series.

Study the inverse problem of finding the right - hand side of a fractional-order subdiffusion equation. We remind you that in this part we assume that the function on the right side of the equation does not depend on the variable t.

Inverse problems for determining the right-hand side (heat source density) of various subdiffusion equations have been studied by many scientists . Full information about this can be found in the article "On the nonlocal boundary value problems for time-fractional equations" by R. Ashurov and Y. Fayziev. In the article "Determination of fractional order and source term in a fractional subdiffusion equation" by the same authors, research on inverse problems in finding the source function of subdiffusion equations and determining the order of the Riemann-Louisville fractional derivative at the same time is highlighted.

In this article, the solution of the inverse problem of finding the right-hand side of the equation is used to prove the existence and uniqueness theorems using the classical Fourier method.

Consider the following inverse problem:

$$D_t^{\rho}u(x,t)-a^2u_{xx}(x,t)=f(x), \qquad 0<x<l, \quad 0<t<T, \quad (2.1)$$

$$u(x,+0)=\varphi(x), \qquad 0\le x\le l, \quad (2.2)$$

$$u(0,t)=0, \qquad 0\le t\le T, \quad (2.3)$$

$$u(l,t)=0, \qquad 0\le t\le T, \quad (2.4)$$

Let's assume that in problem (2.1) - (2.4). $u(x,t)$ let the function $f(x)$ be unknown as well . To solve this problem we need an additional condition . We get the following condition as an additional condition :

$$u(x,\tau)=\psi(x), \qquad 0<\tau<T. \quad (2.5)$$

In this problem (2.1) – (2.5). $u(x,t)$ and $f(x)$ the problem of finding functions is called ***the inverse problem*** of finding the right side of the equation.

2.2 – definition . If $u(x,t)\in C\left([0,l]\times[0,T]\right)$ and $f(x)$ functions $D_t^\rho u(x,t), u_{xx}(x,t)\in C\left((0,l)\times(0,T\right)$ having the properties (2.1) - (2.5) conditions satisfies, then $\{u(x,t), f(x)\}$ the pair of functions (2.1) - (2.5) ***solution of the inverse problem*** is called

Theorem 2.2. let's say $\varphi(x)\in C[0,l]$, has a piecewise-continuous derivative $\psi(x)\in C^2[0,l]$, has a second-order piecewise-continuous derivative, , $\varphi(0)=\varphi(l)=0$ and $\psi''(0)=\psi''(l)=0$ are functions satisfying the conditions , then the inverse problem (2.1) - (2.5) $\{u(x,t), f(x)\}$ has a unique solution , and this The solution looks like this :

$$u(x,t)=\sum_{n=1}^{\infty}\left[\varphi_n E_{\rho,1}\left(-\left(\frac{\pi na}{l}\right)^2 t^\rho\right)+f_n t^\rho E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2 t^\rho\right)\right]\sin\frac{\pi nx}{l},$$

this on the ground

$$f_n=\frac{\psi_n}{\tau^\rho E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)}-\frac{\varphi_n E_{\rho,1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)}{\tau^\rho E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)} \qquad (2.6)$$

and

$$f(x)=\sum_{n=1}^{\infty} f_n\sin\frac{\pi nx}{l} \qquad .(2.7)$$

Proof **.** If $f(x)$ function known function that assumption if we do , he to Theorem 1.1 according to (2.1) - (2.4) matter (1.6) in the form to the solution have will be (2.5) from the condition if we use the following to equality have we will be :

$$u(x,\tau)=\sum_{n=1}^{\infty}\left[\varphi_n E_{\rho,1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)+\right.$$

$$\left.+f_n\tau^\rho E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)\right]\sin\frac{\pi nx}{l}=\psi(x).$$

$\psi(x)$ expanding the function by sines into a Fourier series and equating the corresponding coefficients

$$\varphi_n E_{\rho,1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)+f_n\tau^\rho E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)=\psi_n$$

we will have equality . From this equation

$$f_n\tau^\rho E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)=\psi_n-\varphi_n E_{\rho,1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)$$

we form the equation. From this,

$$f_n=\frac{\psi_n}{\tau^\rho E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)}-\frac{\varphi_n E_{\rho,1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)}{\tau^\rho E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)}$$

it follows that Now $f(x)=\sum_{n=1}^{\infty} f_n\sin\frac{\pi nx}{l}$ we prove that the series is flat convergent. For this, we define the partial sum $F_j(x)=\sum_{n=1}^{j} f_n\sin\frac{\pi nx}{l}$ of this line as According to the above equation

$$F_j(x)=\sum_{n=1}^{j}\left[\frac{\psi_n}{\tau^\rho E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)}-\frac{\varphi_n E_{\rho,1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)}{\tau^\rho E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)}\right]\sin\frac{\pi nx}{l}$$

will be If

$$F_j^1(x)=-\sum_{n=1}^{j}\frac{\varphi_n E_{\rho,1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)}{\tau^\rho E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^\rho\right)}\sin\frac{\pi nx}{l}$$

and

$$F_j^2(x)(x)=\sum_{n=1}^{j}\frac{\psi_n}{\tau^{\rho}E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^{\rho}\right)}\sin\frac{\pi nx}{l}$$

If we include the definitions, then

$$F_j(x)=F_j^1(x)+F_j^2(x)$$

we form the equation. Now let's examine each of these contributors . of the Mittag–Leffler function

$$E_{\rho,\mu}(-t)=t^{-1}+O(t^{-2})$$

Using the asymptotic estimate, we have the following

$$F_j^1(x)=-\sum_{n=1}^{j}\frac{\varphi_n\left(\frac{\pi na}{l}\right)^{-2}\tau^{-\rho}\left(1+O\left(\frac{\pi na}{l}\right)^{-2}\tau^{-\rho}\right)}{\tau^{\rho}\left(\frac{\pi na}{l}\right)^{-2}\tau^{-\rho}\left(1+O\left(\frac{\pi na}{l}\right)^{-2}\tau^{-\rho}\right)}\sin\frac{\pi nx}{l}=$$

$$=-\sum_{n=1}^{j}\frac{\varphi_n}{\tau^{\rho}}\sin\frac{\pi nx}{l}\left(1+O\left(\frac{\pi na}{l}\right)^{-2}\tau^{-\rho}\right)$$

From this, if $\varphi(x)$ the function has a continuous, piecewise -continuous derivative and $\varphi(0)=\varphi(l)=0$ is a function that satisfies the conditions , then the series is flat approximant .

Now we will learn that the second adder is a flat adder .

$$F_j^2(x)(x)=\sum_{n=1}^{j}\frac{\psi_n}{\tau^{\rho}\left(\frac{\pi na}{l}\right)^{-2}\tau^{-\rho}\left(1+O\left(\frac{\pi na}{l}\right)^{-2}\tau^{-\rho}\right)}\sin\frac{\pi nx}{l}=$$

$$=C\sum_{n=1}^{j}\frac{n^2\psi_n}{1+O\left(\frac{\pi na}{l}\right)^{-2}\tau^{-\rho}}\sin\frac{\pi nx}{l}$$

can be seen that if , $\psi(x)\in C^2[0,l]$ has a piecewise-continuous derivative of the second order , $\psi(0)=\psi(l)=0$ and $\psi''(0)=\psi''(l)=0$ is a function satisfying the conditions , then the above series is a smooth approximation .

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Thus, the series of Fourier coefficients (2.6) determined by formulas (2.7) is a smooth approximation .

If the function defined by the series (2.7) is known $f(x)$, we $u(x,t)$ can find the function based on the previous paragraph. Thus, theorem (2.1) is proved. If we simplify and use an additional condition, it will be as follows.

function of finding the right side of an equation $f(x)$ we study for the case where t o ' does not depend on the variable . Then, based on the above paragraph, i.e. (1.1) - theorem, the solution of the correct problem (2.1) - (2.4) will be as follows.

$$u(x,t)=\sum_{n=1}^{\infty}\left[\varphi_n E_{\rho,1}\left(-\left(\frac{\pi na}{l}\right)^2 t^{\rho}\right)+f_n t^{\rho}E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2 t^{\rho}\right)\right]\sin\frac{\pi nx}{l},$$

here

$$f_n=\frac{\psi_n}{\tau^{\rho}E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^{\rho}\right)}-\frac{\varphi_n E_{\rho,1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^{\rho}\right)}{\tau^{\rho}E_{\rho,\rho+1}\left(-\left(\frac{\pi na}{l}\right)^2\tau^{\rho}\right)} \qquad (2.6)$$

and

$$f(x)=\sum_{n=1}^{\infty} f_n \sin\frac{\pi nx}{l} \qquad .(2.7)$$

(2.6) to the solution (2.1) - (2.5) *solution of the inverse problem* is called

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